Syllabus of Electricity and Magnetism

1. Fundamentals of Classical Electromagnetism.
2. Electrostatics:
   a) Charges.
   b) Electrostatic Force.
   c) Electric Field.
   d) Electric Flux.
   e) Gausses Law.
   f) Electric Potential.
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3. Magnetostatics:
   a) Magnetic Field.
   b) Magnetic Flux.
   c) Electric current and Amperes Low.
   d) Faradays Low of Electromagnetic induction.
4. Maxwell equations:
   a) Electromagnetic waves and the nature of light.

References:

1. Physics for scientist and engineers by Raymond Serway and John Jewett, 9th Ed.
2. Electricity and Magnetism by David Morin and Edward Purcell, 3rd Ed.

Fundamentals of classical electromagnetism

Electromagnetism is a branch of Physics that deals with the electromagnetic force that occurs between electrically charged particles. The electromagnetic force is one of the four fundamental forces and exhibits electromagnetic fields such as magnetic fields, electric fields, and light. It is the basic reason electrons bound to the nucleus and responsible for the complete structure of the nucleus.

The electromagnetic force is a type of physical interaction that occurs between electrically charged particles. It acts between charged particles and is the combination of all magnetic and electrical forces. The electromagnetic force can be attractive or repulsive.

Before the invention of electromagnetism, people or scientists used to think electricity and magnetism are two different topics. The view has changed after James Clerk Maxwell published **A Treatise on Electricity and Magnetism** in the year 1873. The publication states that the interaction of positive and negative charges are mediated by one force. This observation laid a foundation for Electromagnetism.

Electrostatics is the study of electromagnetic phenomena that occur when there are no moving charges (at rest) —i.e., after a static equilibrium has been established. Charges reach their equilibrium positions rapidly, because the electric force is extremely strong.

Electric Charges

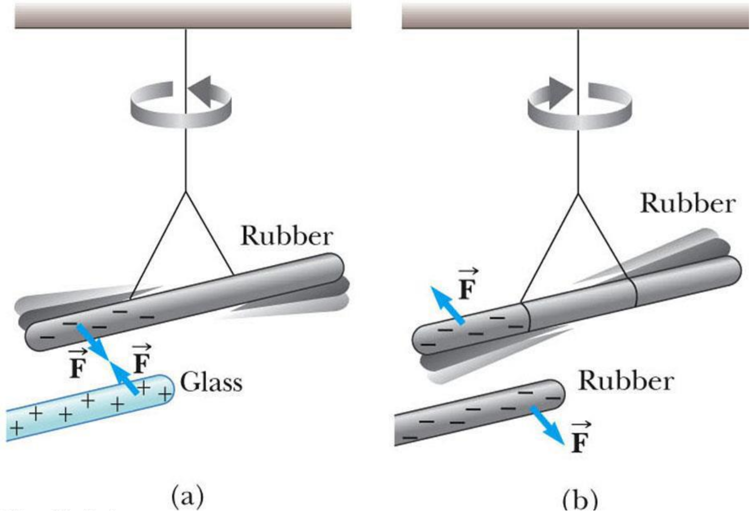
<u>Experiments</u>

1- After running a comb through your hair on a dry day you will find that the comb attracts bits of paper.
2- Certain materials are rubbed together, such as glass rubbed with silk or rubber with fur, same effect will appear.
3- Another simple experiment is to rub an inflated balloon with wool. The balloon then adheres to a wall, often for hours.

Results

When materials behave in this way, they are said to be electrified, or to have become electrically charged.

- There are two kinds of electric charges: positive and negative.
  - Negative charges are the type possessed by electrons.
  - Positive charges are the type possessed by protons.
- Charges of the same sign **repel** one another.
- Charges with opposite signs **attract** one another.

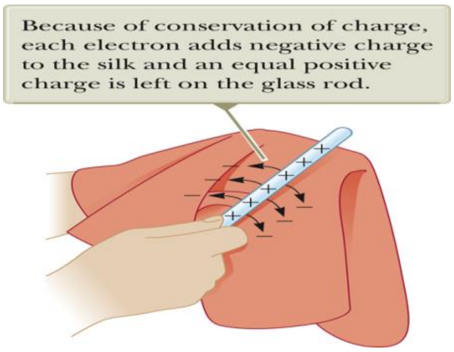


(a) The rubber rod is negatively charged and the glass rod is positively charged. The two rods will **attract**.

(b) The rubber rod is negatively charged and the second rubber rod is also negatively charged. The two rods will **repel**.

- Electric charge is always **conserved in an isolated system**.
  - For example, charge is not created in the process of rubbing two objects together.
  - The electrification is due to a **transfer of charge from one object to another**.

Conservation of Electric Charges



- A glass rod is rubbed with silk.
- Electrons are transferred from the glass to the silk.
- Each electron adds a negative charge to the silk.
- An equal positive charge is left on the rod.

The law of conservation of charge states that electric charge can neither be created nor destroyed. In a closed system, the amount of charge remains the same. When something changes its charge it doesn't create charge but transfers it.

Quantization of Electric Charges

Quantization of charge means that when we say something has a given charge, we mean that that is how many times the charge of a single electron it has. Because all charges are associated with a whole electron, this is possible.

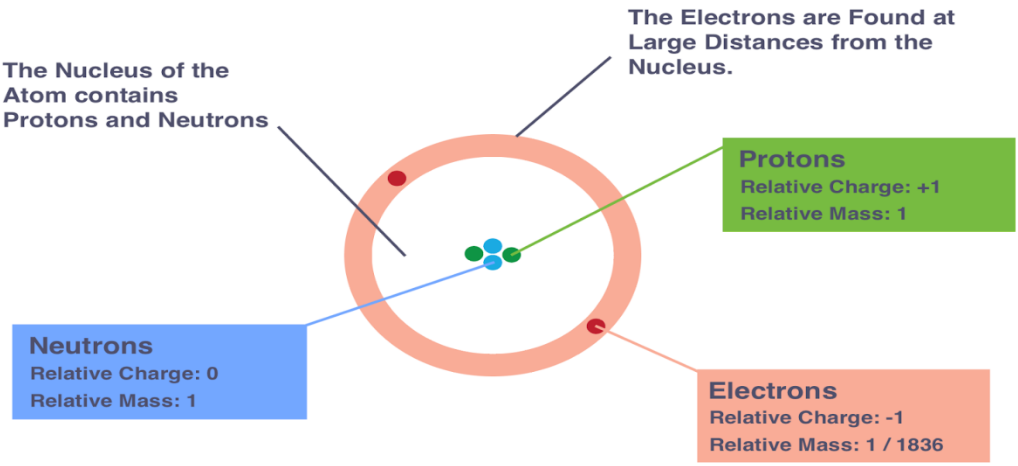
The electric charge is said to be **quantized**.

- q is the standard symbol used for charge as a variable.
- Electric charge exists as discrete packets.

$$q = \pm Ne \qquad N \text{ is an integer}$$

e is the fundamental unit of charge

- $|e| = 1.6 \times 10^{-19}$ C
- Electron: $q = -e$
- Proton: $q = +e$



Conductors, Insulators and Semiconductors

Can be classifying materials in terms of the ability of electrons to move through the material:

<u>Conductors</u>: Electrical conductors are materials in which most of the electrons are free electrons.

- Free electrons are not bound to the atoms.
- These electrons can move relatively freely through the material.
- Examples of good conductors include copper, aluminum and silver.
- When a good conductor is charged in a small region, the charge readily distributes itself over the entire surface of the material.

<u>Insulators</u>: Electrical insulators are materials in which all of the electrons are bound to atoms.

- These electrons cannot move relatively freely through the material.
- Examples of good insulators include glass, rubber and wood.
- When a good insulator is charged in a small region, the charge is unable to move to other regions of the material.

<u>Semiconductors</u>: The electrical properties of semiconductors are somewhere between those of insulators and conductors.

- Examples of semiconductor materials include silicon and germanium.
- Semiconductors made from these materials are commonly used in making electronic chips.
- The electrical properties of semiconductors can be changed by the addition of controlled amounts of certain atoms to the material.

Charging by Induction

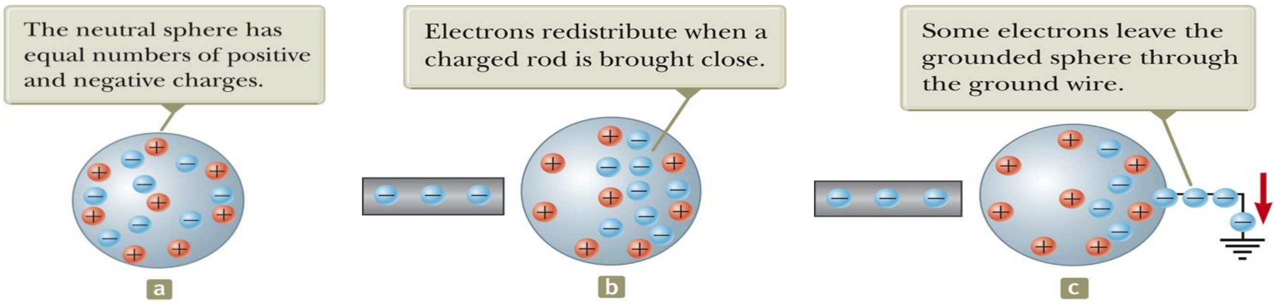
Charging by induction requires no contact with the object inducing the charge. Assume we start with a neutral metallic sphere.

(a) The sphere has the same number of positive and negative charges.

(b) A charged rubber rod is placed near the sphere.

- It does not touch the sphere.
- The electrons in the neutral sphere are redistributed.

(c) The sphere is grounded. Some electrons can leave the sphere through the ground wire.

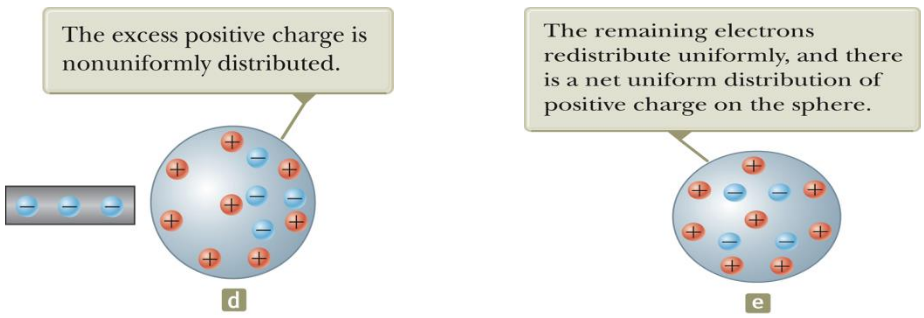


(d) The ground wire is removed.

- There will now be more positive charges.
- The charges are not uniformly distributed.
- The positive charge has been induced in the sphere.

(e) The rod is removed.

- The electrons remaining on the sphere redistribute themselves.
- There is still a net positive charge on the sphere.
- The charge is now uniformly distributed.
- Note the rod lost none of its negative charge during this process.



Coulomb's Law

Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented. Consider a system of two point charges, q_1 and q_2, separated by a distance r in vacuum.

He found the force depended on the charges and the distance between them. The electric force between two stationary charged particles:

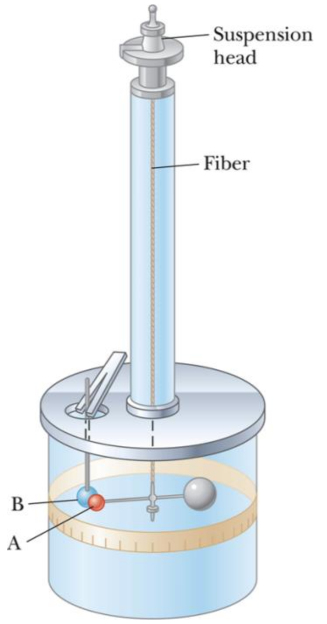
- Is <u>inversely</u> proportional to the <u>square of the separation</u> r between the particles and directed along the line joining them.

$$F_e \propto 1/r^2$$

- Is <u>proportional</u> to the <u>product</u> of the charges q_1 and q_2 on the two particles.

$$F_e \propto q_1 q_2$$

- Is <u>attractive</u> if the charges are of <u>opposite</u> sign and <u>repulsive</u> if the charges have the <u>same</u> sign.
- Is a <u>conservative</u> force.



Coulomb's torsion balance, used to establish the inverse-square law for the electric force between two charges.

Coulomb's Law, Equation

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

Where k_e is the **Coulomb constant**

- $k_e = 1/(4\pi \varepsilon_o) = 9 \times 10^9$ N.m^2/C^2

Where ε_o is the **permittivity of free space**

- $\varepsilon_o = 8.8542 \times 10^{-12}$ C^2 / N.m^2

- The smallest unit of charge known in nature is the charge on an electron or proton, which has an absolute value of
  - $e = 1.602 \times 10^{-19}$ C

- 1 C of charge is approximately equal to the charge of 6.24 x 10^{18} electrons or protons.
- The force is a vector quantity.

Charge and Mass of Electron, Proton and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,176\,5 \times 10^{-19}$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,5 \times 10^{-19}$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

- The electron and proton are identical in the magnitude of their charge, but very different in mass.
- The proton and the neutron are similar in mass, but very different in charge.

Example 1: The Hydrogen Atom

The electron and proton of a hydrogen atom are separated by a distance of approximately 5.3 x 10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution: From Coulomb's law, we find that the attractive electric force has the magnitude:

$$q_1 = -1.6 \times 10^{-19}\ \text{C} \quad \text{and} \quad q_2 = +1.6 \times 10^{-19}\ \text{C}$$

$$F_e = k_e \frac{|e|^2}{r^2} = \left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2} = 8.2 \times 10^{-8}\ \text{N}$$

Using Newton's law of gravitation, we find that the gravitational force has the magnitude

$$F_g = G\frac{m_e m_p}{r^2}$$

$$= \left(6.7 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2} = 3.6 \times 10^{-47}\ \text{N}$$

The ratio $F_e / F_g \approx 2 \times 10^{39}$.

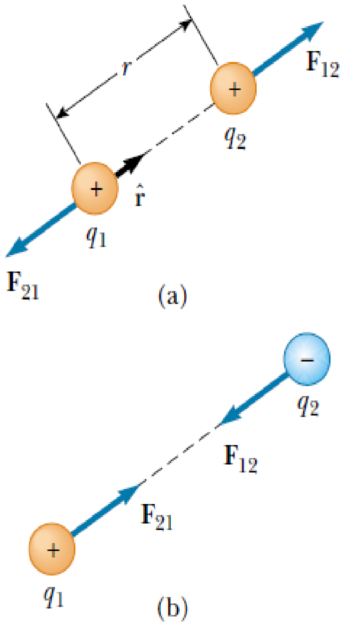
Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force.

Vector Nature of Electric Forces

When dealing with Coulomb's law, you must remember that force is a vector quantity. Thus, the law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2 , written F_{12} , is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

Where $\hat{r}$ *is a unit vector directed from* q_1 to q_2



- **Electrical forces obey Newton's Third Law:** the force on q_1 is equal in magnitude and opposite in direction to the force on q_2

$$\vec{F}_{21} = -\vec{F}_{12}$$

- The like charges produce a repulsive force between them .
- With like signs for the charges, the product q_1q_2 is positive and the force is repulsive.

Two point charges are separated by a distance r.

- The unlike charges produce an attractive force between them.
- With unlike signs for the charges, the product q_1q_2 is negative and the force is attractive.
  - **When more than two charges** are present, the force between any pair of them is given by Equation 2. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is

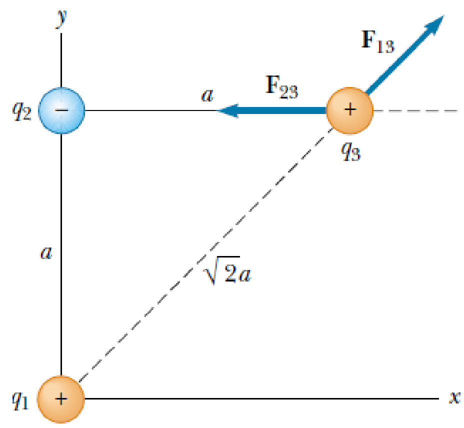
$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

Example 2: Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure, where $q_1 = q_3 = 5\mu C$, $q_2 = -2\mu C$, and a = 0.10 m. Find the resultant force exerted on q_3.

Solution: The force exerted by q_1 on q_3 is $\mathbf{F_{13}}$. The force exerted by q_2 on q_3 is $\mathbf{F_{23}}$. The resultant force F_3 exerted on q_3 is the vector sum $\mathbf{F_T}=\mathbf{F_{13}} + \mathbf{F_{23}}$.

The magnitude of F_{23} is



$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$
$$= \left(8.99 \times 10^9 \frac{N\cdot m^2}{C^2}\right) \frac{(2.0 \times 10^{-6} C)(5.0 \times 10^{-6} C)}{(0.10 m)^2}$$
$$= 9.0 N$$

Note that because q_3 and q_2 have opposite signs, F_{23} is to the left.

The magnitude of the force exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$
$$= \left(8.99 \times 10^9 \frac{N\cdot m^2}{C^2}\right) \frac{(5.0 \times 10^{-6} C)(5.0 \times 10^{-6} C)}{2(0.10 m)^2}$$
$$= 11 N$$

The repulsive force $\mathbf{F_{13}}$ makes an angle of 45° with the x axis. Therefore, the x and y components of F_{13} are equal, with magnitude given by $\mathbf{F_{13} \cos 45° = 7.9 N}$.

$$\mathbf{F_{13x} = F_{13} \cos 45° = 7.9 N.}$$

$$\mathbf{F_{13y} = F_{13} \sin 45° = 7.9 N.}$$

$$\mathbf{F_{3x} = F_{13x} + F_{23x} = 7.9 N + (-9.0 N) = -1.1 N}$$

$$\mathbf{F_{3y} = F_{13y} + F_{23y} = 7.9 N + 0 = 7.9 N}$$

We can also express the resultant force acting on q_3 in unit-vector form as

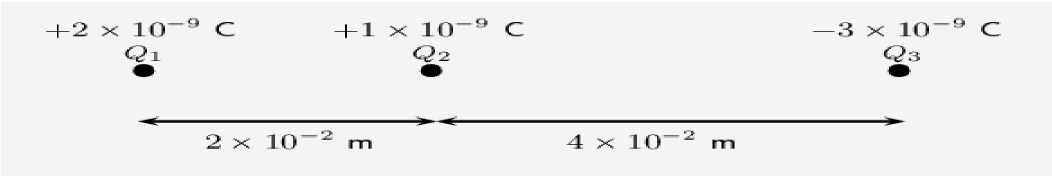
$$\mathbf{F_3} = (-1.1\mathbf{i} + 7.9\mathbf{j}) N$$

The angle that the force makes with the positive x-axis is

$$\phi = \tan^{-1}\left(\frac{7.9}{-1.1}\right) = 98^\circ$$

Example 3:

Three point charges are in a straight line. Their charges are Q_1=+2×10^{-9} C, Q_2=+1×10^{-9} C and Q_3=−3×10^{-9} C. The distance between Q_1 and Q_2 is 2×10^{-2} m and the distance between Q_2 and Q_3 is 4×10^{-2} m. What is the net electrostatic force on Q_2 due to the other two charges?



Solution:

Force on Q_2 due to Q_1:

$$\begin{aligned} F_1 &= k\frac{Q_1Q_2}{r^2} \\ &= (9{,}0 \times 10^9)\frac{(2 \times 10^{-9})(1 \times 10^{-9})}{(2 \times 10^{-2})^2} \\ &= (9{,}0 \times 10^9)\frac{(2 \times 10^{-9})(1 \times 10^{-9})}{(4 \times 10^{-4})} \\ &= 4{,}5 \times 10^{-5}\ \text{N} \end{aligned}$$

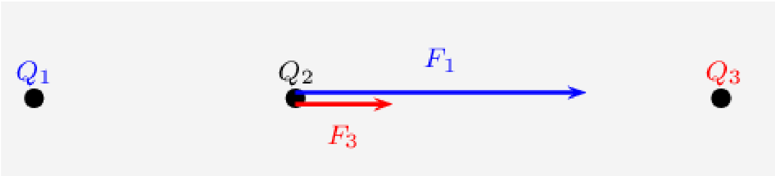
Force on Q_2 due to Q_3:

$$\begin{aligned} F_3 &= k\frac{Q_2Q_3}{r^2} \\ &= (9{,}0 \times 10^9)\frac{(1 \times 10^{-9})(3 \times 10^{-9})}{(4 \times 10^{-2})^2} \\ &= (9{,}0 \times 10^9)\frac{(1 \times 10^{-9})(3 \times 10^{-9})}{(16 \times 10^{-4})} \\ &= 1{,}69 \times 10^{-5}\ \text{N} \end{aligned}$$

**** Vector addition of forces**

The force between Q_1 and Q_2 is repulsive (like charges). This means that it pushes Q_2 to the right, or in the positive direction.

The force between Q_2 and Q_3 is attractive (unlike charges) and pulls Q_2 to the right.



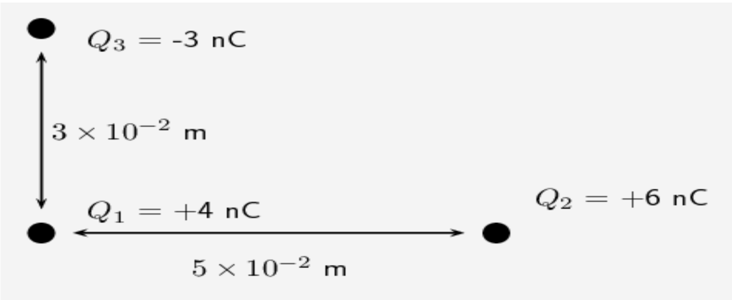
Therefore both forces are acting in the positive direction. Therefore,

$$\begin{aligned} F_R &= 4{,}5 \times 10^{-5}\ \mathrm{N} + 1{,}69 \times 10^{-5}\ \mathrm{N} \\ &= 6{,}19 \times 10^{-5}\ \mathrm{N} \end{aligned}$$

The resultant force acting on Q_2 is $6{,}19\times10^{-5}$ N to the right.

Example 4:

Three point charges form a right-angled triangle. Their charges are Q_1= 4 nC, Q_2= 6 nC and Q_3= −3 nC. The distance between Q_1 and Q_2 is 5×10^{-2} m and the distance between Q_1 and Q_3 is 3×10^{-2} m. What is the net electrostatic force on Q_1 due to the other two charges if they are arranged as shown?



Solution:

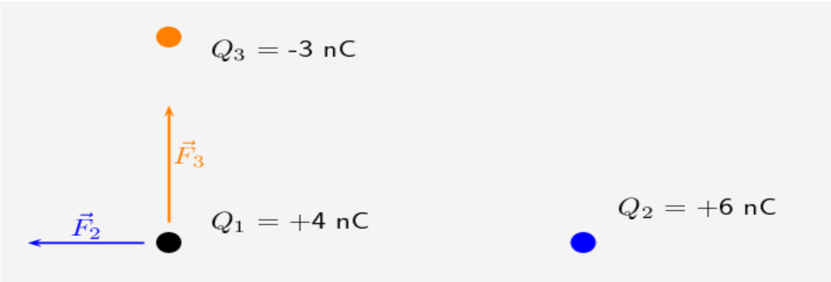
The magnitude of the force exerted by Q_2 on Q_1, which we will call F_2, is:

$$\begin{aligned} F_2 &= k\frac{Q_1Q_2}{r^2} \\ &= (9{,}0 \times 10^9)\frac{(4 \times 10^{-9})(6 \times 10^{-9})}{(5 \times 10^{-2})^2} \\ &= (9{,}0 \times 10^9)\frac{(4 \times 10^{-9})(6 \times 10^{-9})}{(25 \times 10^{-4})} \\ &= 8{,}630 \times 10^{-5}\ \mathrm{N} \end{aligned}$$

The magnitude of the force exerted by Q_3 on Q_1, which we will call F_3, is:

$$
\begin{aligned}
F_3 &= k\frac{Q_1Q_3}{r^2} \\
&= (9{,}0 \times 10^9)\frac{(4 \times 10^{-9})(3 \times 10^{-9})}{(3 \times 10^{-2})^2} \\
&= (9{,}0 \times 10^9)\frac{(4 \times 10^{-9})(3 \times 10^{-9})}{(9 \times 10^{-4})} \\
&= 1{,}199 \times 10^{-4}\ \text{N}
\end{aligned}
$$

** Vector addition of forces



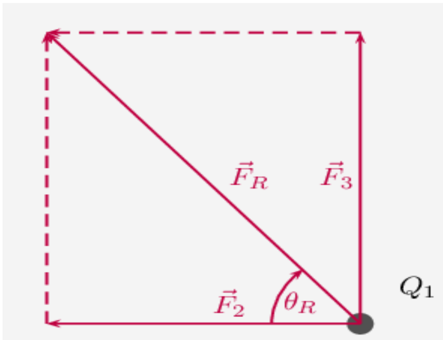
** Resultant force

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the *x*- and *y*-directions:

$$
\begin{aligned}
F_R^2 &= F_2^2 + F_3^2 \text{by Pythagoras' theorem} \\
F_R &= \sqrt{(8{,}630 \times 10^{-5})^2 + (1{,}199 \times 10^{-4})^2} \\
F_R &= 1{,}48 \times 10^{-4}\ \text{N}
\end{aligned}
$$

and the angle, θ_R made with the *x*-axis can be found using trigonometry.

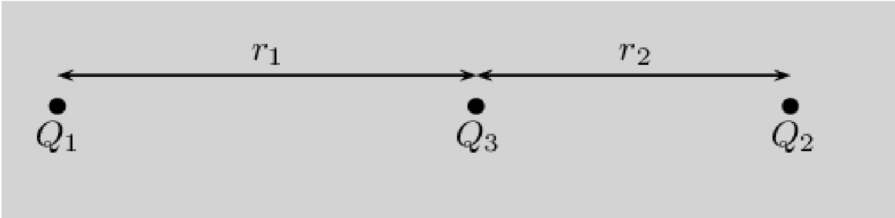
$$
\begin{aligned}
\tan(\theta_R) &= \frac{\text{y-component}}{\text{x-component}} \\
\tan(\theta_R) &= \frac{1{,}199 \times 10^{-4}}{8{,}630 \times 10^{-5}} \\
\theta_R &= \tan^{-1}\left(\frac{1{,}199 \times 10^{-4}}{8{,}630 \times 10^{-5}}\right) \\
\theta_R &= 54{,}25^\circ \text{ to 2 decimal places}
\end{aligned}
$$



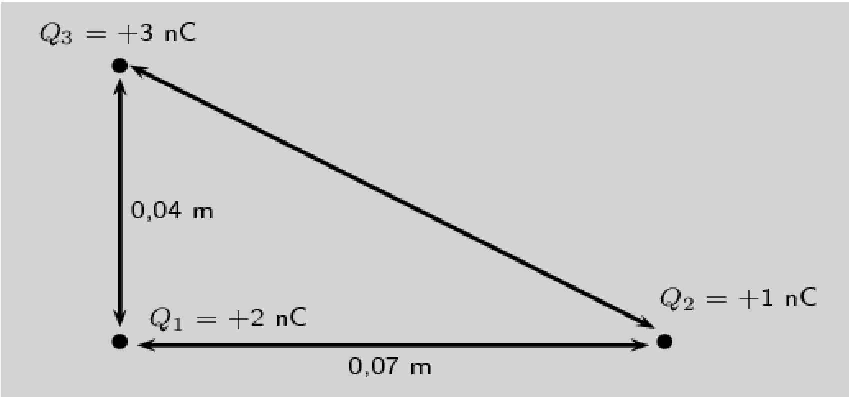
The final resultant force acting on Q_1 is $1{,}48\times10^{-4}$ N acting at an angle of 54,25° to the negative *x*-axis or 125,75°to the positive *x*-axis.

Problem 1: For the charge configuration shown, calculate the resultant force on Q_2 if:

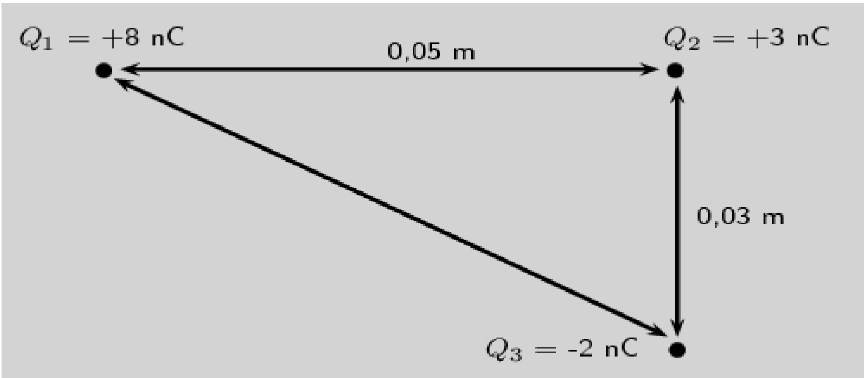
- Q_1=2,3 × 10^{-7} C
- Q_2=4 × 10^{-6} C
- Q_3=3,3 × 10^{-7} C
- r_1=2,5 × 10^{-1} m
- r_2=3,7 × 10^{-2} m



Problem 2: Calculate the resultant force on Q_1 given this charge configuration:

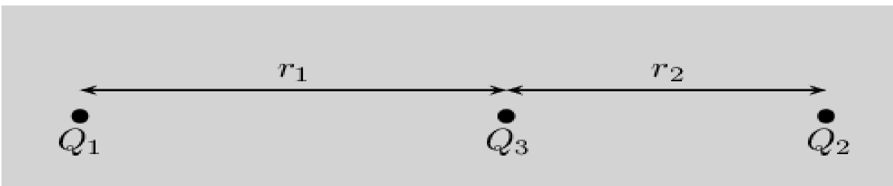


Problem 3: Calculate the resultant force on Q_2 given this charge configuration:



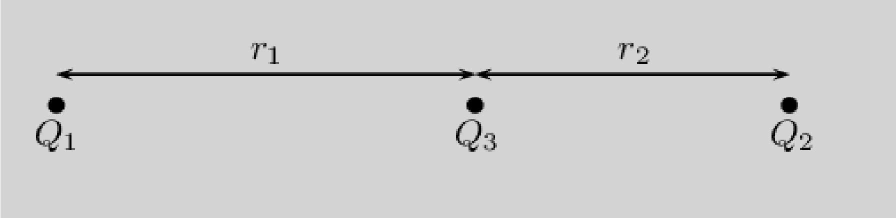
Problem 4: For the charge configuration shown, calculate the charge on Q_3 if the resultant force on Q_2 is 6,3×10^{-1} N to the right and:

- Q_1=4,36 × 10^{-6} C
- Q_2=−7 × 10^{-7} C
- r_1=1,85 × 10^{-1} m
- r_2=4,7 × 10^{-2} m



Problem 1: For the charge configuration shown, calculate the resultant force on Q_2 if:

- Q_1=$2{,}3 \times 10^{-7}$ C
- Q_2=4×10^{-6} C
- Q_3=$3{,}3 \times 10^{-7}$ C
- r_1=$2{,}5 \times 10^{-1}$ m
- r_2=$3{,}7 \times 10^{-2}$ m



Solution:

We first calculate the force of Q_1 on Q_2. Note that for this force we must add r_1 and r_2.

$$\begin{aligned} F_{e1} &= \frac{kQ_1Q_2}{r^2} \\ &= \frac{(9{,}0 \times 10^9)(2{,}3 \times 10^{-7})(4 \times 10^{-6})}{(3{,}7 \times 10^{-2} + 2{,}5 \times 10^{-1})^2} \\ &= 1{,}00 \times 10^{-1}\ \text{N} \end{aligned}$$

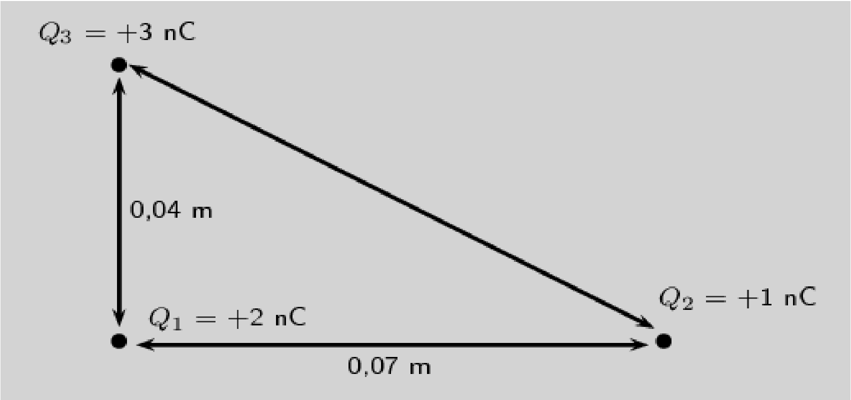
And then we calculate the force on Q_2 from Q_3:

$$\begin{aligned} F_{e2} &= \frac{kQ_2Q_3}{r^2} \\ &= \frac{(9{,}0 \times 10^9)(4 \times 10^{-6})(3{,}3 \times 10^{-7})}{(\ 3{,}7 \times 10^{-2})^2} \\ &= 8{,}67\ \text{N} \end{aligned}$$

Next we note that the force of Q_3 on Q_2 is repulsive and the force of Q_1 on Q_2 is also repulsive. So these two forces act in the same direction (towards the right). The resultant force is:

$$\begin{aligned} F_{eR} &= F_{e1} + F_{e2} \\ &= 8{,}67\ \text{N} + 0{,}1\ \text{N} \\ &= 8{,}77\ \text{N}\ \textit{to the right.} \end{aligned}$$

Problem 2: Calculate the resultant force on Q_1 given this charge configuration:



Solution:

We first calculate the force on Q_1 from Q_2:

$$\begin{aligned} F_e &= \frac{kQ_1Q_2}{r^2} \\ &= \frac{(9{,}0 \times 10^9)(1 \times 10^{-9})(2 \times 10^{-9})}{(0{,}07)^2} \\ &= 3{,}7 \times 10^{-6} \text{ N} \end{aligned}$$

And then we calculate the force of Q_3 on Q_1:

$$\begin{aligned} F_e &= \frac{kQ_1Q_3}{r^2} \\ &= \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})(2 \times 10^{-9})}{(0{,}04)^2} \\ &= 3{,}4 \times 10^{-5} \text{ N} \end{aligned}$$

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

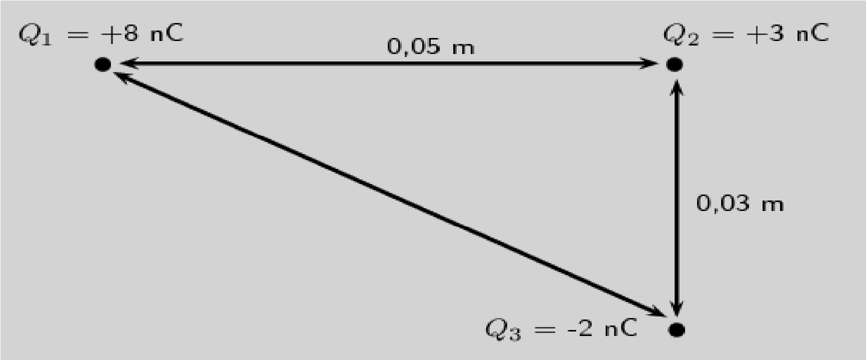
$$\begin{aligned} F_R^2 &= F_x^2 + F_y^2 \\ F_R &= \sqrt{(3{,}7 \times 10^{-6})^2 + (3{,}4 \times 10^{-5})^2} \\ &= 3{,}42 \times 10^{-5} \text{ N} \end{aligned}$$

We can find the angle using trigonometry:

$$\tan\theta_R = \frac{y\text{-component}}{x\text{-component}} = \frac{3{,}42 \times 10^{-5}}{3{,}7 \times 10^{-6}} = 9{,}2432\ldots$$
$$\theta_R = 83{,}8^\circ$$

The final resultant force acting on Q_1 is $3{,}42\times10^{-5}$ N acting at an angle of 83,8° to the negative x-axis.

Problem 3: Calculate the resultant force exerted on Q_2 given this charge configuration:



Solution:

We first calculate the force on Q_2 from Q_1:

$$F_e = \frac{kQ_1Q_2}{r^2} = \frac{(9{,}0 \times 10^9)(8 \times 10^{-9})(3 \times 10^{-9})}{(0{,}05)^2} = 8{,}63 \times 10^{-5}\ \text{N}$$

And then we calculate the force of Q_3 on Q_2:

$$F_e = \frac{kQ_2Q_3}{r^2} = \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})(2 \times 10^{-9})}{(0{,}03)^2} = 5{,}99 \times 10^{-5}\ \text{N}$$

The magnitude of the resultant force acting on Q_2 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the *x*- and *y*-directions:

$$F_R^2 = F_x^2 + F_y^2$$
$$F_R = \sqrt{(8{,}63 \times 10^{-5})^2 + (5{,}99 \times 10^{-5})^2}$$
$$= 1{,}05 \times 10^{-4}\ \text{N}$$

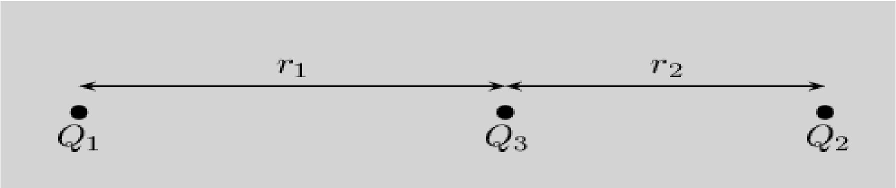
We can find the angle using trigonometry:

$$\tan\theta_R = \frac{y\text{-component}}{x\text{-component}}$$
$$= \frac{5{,}99 \times 10^{-5}}{8{,}63 \times 10^{-5}}$$
$$= 0{,}694\ldots$$
$$\theta_R = 34{,}76^\circ$$

The final resultant force acting on Q_1 is $1{,}05\times10^{-4}$ N acting at an angle of 34,76° to the positive *x*-axis.

Problem 4: For the charge configuration shown, calculate the charge on Q_3 if the resultant force on Q_2 is $6{,}3\times10^{-1}$ N to the right and:

- $Q_1 = 4{,}36 \times 10^{-6}$ C
- $Q_2 = -7 \times 10^{-7}$ C
- $r_1 = 1{,}85 \times 10^{-1}$ m
- $r_2 = 4{,}7 \times 10^{-2}$ m



Solution:

We are told that the resultant force is $6{,}3\times10^{-5}$ N to the right. Since the force of Q_1 on Q_2 is attractive, the force of Q_3 on Q_2 must be repulsive to cause a resultant force to the right (if it was also attractive, the resultant force would be to the left). So we know that Q_3 must be negative.

We first calculate the force on Q_2 from Q_1:

$$\begin{aligned} F_{e1} &= \frac{kQ_1Q_2}{r^2} \\ &= \frac{(9{,}0 \times 10^9)(4{,}36 \times 10^{-6})(7 \times 10^{-7})}{(1{,}85 \times 10^{-1} + 4{,}7 \times 10^{-2})^2} \\ &= 0{,}51 \text{ N} \end{aligned}$$

Next we use this and the resultant force to find the force on Q_2 from Q_3

$$\begin{aligned} F_{eR} &= F_{e1} + F_{e2} \\ F_{e2} &= 6{,}3 \times 10^{-1} \text{ N} - 0{,}51 \text{ N} \\ &= 0{,}12 \text{ N} \end{aligned}$$

And then we calculate the charge on Q_3:

$$\begin{aligned} F_{e2} &= \frac{kQ_2Q_3}{r^2} \\ 0{,}12 &= \frac{(9{,}0 \times 10^9)(7 \times 10^{-7})(Q_3)}{(4{,}7 \times 10^{-2})^2} \\ 2{,}6 \times 10^{-4} &= (6{,}293 \times 10^3)(Q_3) \\ Q_3 &= 4{,}2 \times 10^{-8} \text{ C} \end{aligned}$$

The Electric Field

- The electric force is a field force.
- Field forces can act through space producing effect even with no physical contact between interacting objects.
- An electric field is said to exist in the region of space around a charged object. This charged object is the source charge.
- When another charged object, the test charge, enters this electric field, an electric force acts on it.
- The electric field is defined as the electric force on the test charge per unit charge.

The electric field vector **E** at a point in space is **defined** as the electric force **F** acting on a positive test charge q_0 placed at that point divided by the test charge:

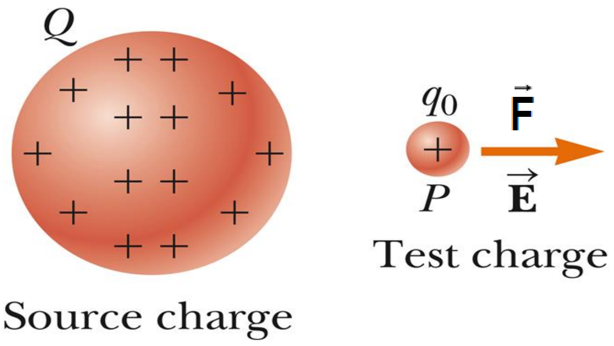
$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{…………………. (1)}$$

The SI units of **E** are N/C.

Note that **E** is the field produced by some charge or charge distribution separate from the test charge; it is not the field produced by the test charge itself.

Also, note that the existence of an electric field is a property of the source charge; the presence of the test charge is not necessary for the field to exist.

- The test charge serves as a detector of the field.



- The direction of **E** is that of the force on a positive test charge.
- We can also say that an electric field exists at a point if a test charge at that point experiences an electric force.

Relationship between F and E

Equation 1 can be rearranged as

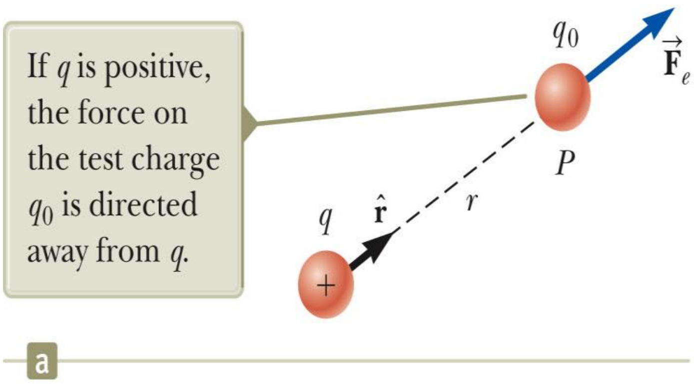
$$\overrightarrow{F} = q\overrightarrow{E} \qquad \text{…………..} (2)$$

This equation gives us the force on a charged particle placed in an electric field.

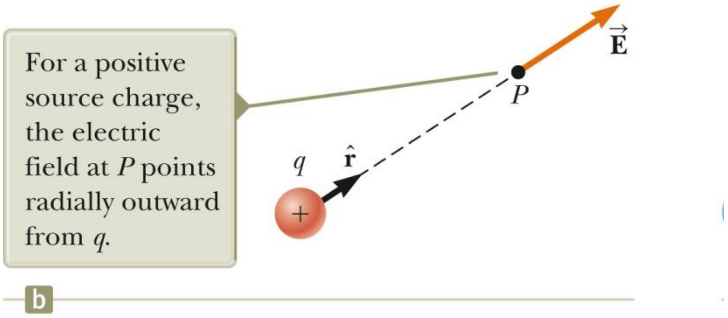
- This is valid for a point charge only.
- For larger objects, the field may vary over the size of the object.
- If source charge, *q*, is positive, the force and the field are in the same direction.
- If source charge, *q*, is negative, the force and the field are in opposite directions.

Electric Field Direction

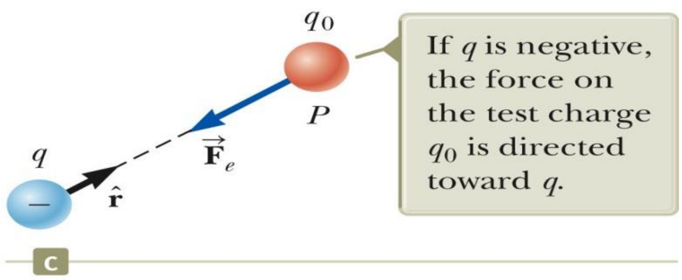
a) If *q* is positive, then the force on the test charge is directed away from *q*.



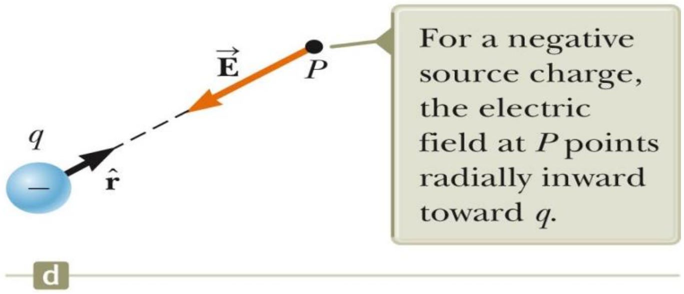
b) The direction of the electric field at P points is also away from the positive source charge.



c) If q is negative, then the force on the test charge is directed toward q.



d) The electric field at P points is also toward the negative source charge.



Electric Field, Vector Form

According to Coulomb's law, the force exerted by source charge q on the test charge q_o, can be expressed as:

$$\vec{F} = K\frac{q\ q_0}{r^2}\ \hat{r}$$

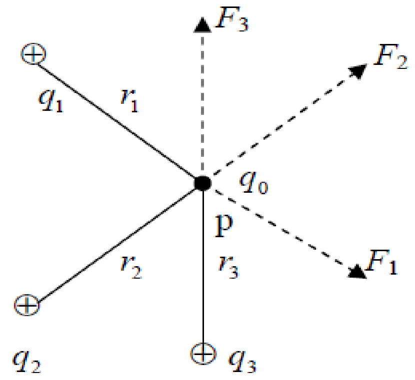
where $\hat{r}$ is a unit vector directed from q toward q_o.

The electric field at P, the position of the test charge is defined by (**E** = $\mathbf{F}_e$ / $\mathbf{q_o}$):

$$\vec{E} = K\frac{q}{r^2}\hat{r} \qquad \text{………………… (3)}$$

Superposition with Electric Fields

At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges, can be expressed



$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r_1^2}\hat{r}_1 \;,\quad \vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_0}{r_2^2}\hat{r}_2 \;,\quad \vec{F}_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_0}{r_3^2}\hat{r}_3$$

$$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots\ldots\ \ldots\ldots\ldots = \sum_i \vec{F}_i$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_0}{r_2^2}\hat{r}_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_0}{r_3^2}\hat{r}_3 + \ldots\ldots\ldots\ \ldots\ldots$$

$$= \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\right)$$

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots\ldots\ldots\ldots\ldots\ldots\right)$$

Or

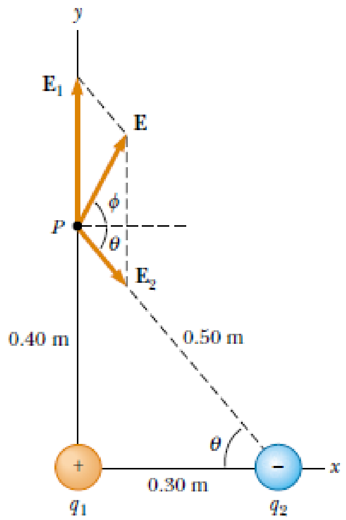
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

Where r_i is the distance from the *i*th source charge q_i to the point P (the location of the test charge) and $\hat{r}_i$ is a unit vector directed from q_i toward point P.

Example 1:

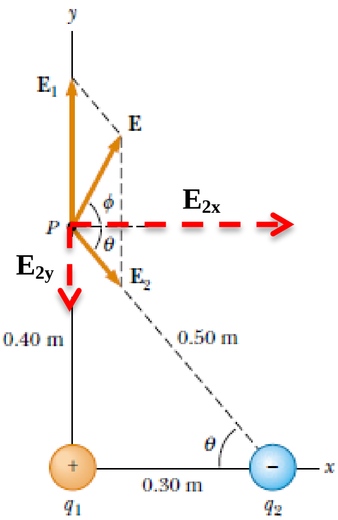
A charge q_1 = 7.0 μC is located at the origin, and a second charge q_2 = -5.0 μC is located on the x axis, 0.30 m from the origin. Find the electric field at the point P, which has coordinates (0, 0.40) m.

Solution:



The total electric field **E** at P equals the vector sum $\mathbf{E_1}+\mathbf{E_2}$, where $\mathbf{E_1}$ is the field due to the positive charge q_1 and $\mathbf{E_2}$ is the field due to the negative charge q_2.

$$E_1 = k_e\frac{|q_1|}{r_1^2} = \left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(7.0\times10^{-6}\,\text{C})}{(0.40\,\text{m})^2}$$
$$= 3.9\times10^5\,\text{N/C}$$
$$E_2 = k_e\frac{|q_2|}{r_2^2} = \left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(5.0\times10^{-6}\,\text{C})}{(0.50\,\text{m})^2}$$
$$= 1.8\times10^5\,\text{N/C}$$



The vector $\mathbf{E_1}$ has only a y component. The vector $\mathbf{E_2}$ has an x component given by $\mathbf{E_2}\cos\theta = 3/5\,\mathbf{E_2}$ and a negative y component given by $-\mathbf{E_2}\sin\theta = -4/5\,\mathbf{E_2}$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9\times10^5\,\mathbf{j}\ \text{N/C}$$
$$\mathbf{E}_2 = (1.1\times10^5\,\mathbf{i} - 1.4\times10^5\,\mathbf{j})\ \text{N/C}$$

The resultant field **E** at P is the superposition of $\mathbf{E_1}$ and $\mathbf{E_2}$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1\times10^5\,\mathbf{i} + 2.5\times10^5\,\mathbf{j})\ \text{N/C}$$

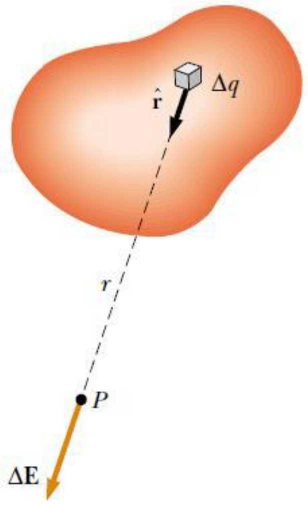
$$\phi = \tan^{-1}\left(\frac{2.5}{1.1}\right) = \mathbf{66^\circ}$$

makes with the positive x axis and it has a magnitude

$$|E| = \sqrt{(1.1\times10^5)^2 + (2.5\times10^5)^2} = 2.7\times10^5\ \text{N/C}$$

Electric Field of a Continuous Charge Distribution

The electric field at *P* due to one charge element carrying charge Δq is



$$\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

The total electric field $\Delta \mathbf{E}$ at *P* due to all elements in the charge distribution is approximately

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Because the charge distribution is modeled as continuous, the total field at *P* in the limit $\Delta q_i \longrightarrow 0$ is

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

We illustrate this type of calculation with several examples, in which we assume the charge is uniformly distributed on a line, on a surface, or throughout a volume.

- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density** ρ is defined by

$$\rho = \frac{Q}{V} \quad (\text{C/m}^3)$$

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density** σ is defined by

$$\sigma = \frac{Q}{A} \quad (\text{C/m}^2)$$

- If a charge Q is uniformly distributed along a line of length ℓ , the **linear charge density** λ is defined by

$$\lambda = \frac{Q}{\ell} \quad (\text{C/m})$$

- If the charge is non-uniformly distributed over a volume, surface, or line, the amounts of charge dq in a small volume, surface, or length element are

$$dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$$

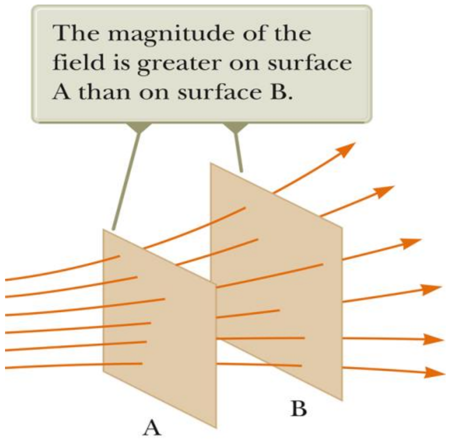
Electric Field Lines

They are an imaginary line drawn through a region of space so that, at every point, it is tangent to the direction of the electric field vector at that point.

The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.

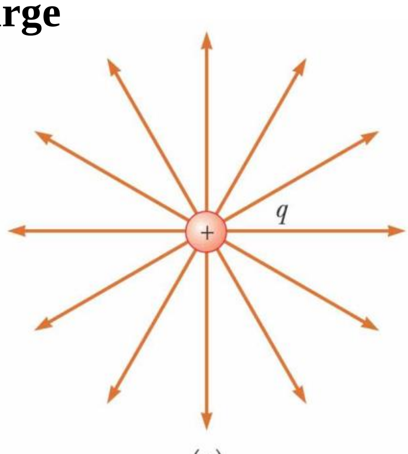
- The density of lines through surface A is greater than the density of lines through surface B.
- The magnitude of the electric field is larger on surface A than on surface B.
- The lines at different locations point in different directions.
- This indicates the field is nonuniform.



The electric field lines for a point charge

a) The electric field lines for a Positive Point Charge

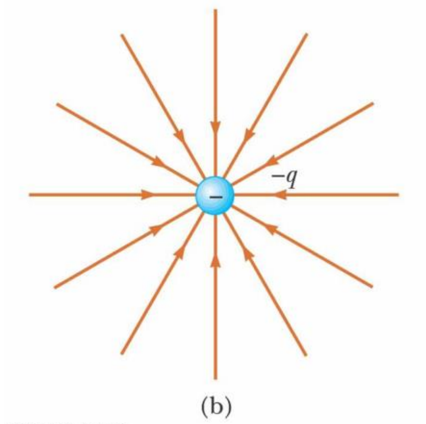
- The field lines radiate outward in all directions.
- In three dimensions, the distribution is spherical.
- The lines are directed away from the source charge.
- A positive test charge would be repelled away from the positive source charge.



(a)

b) The electric field lines for a Negative Point Charge

- The field lines radiate inward in all directions.
- In three dimensions, the distribution is spherical.
- The lines are directed toward the source charge.
- A positive test charge would be attracted toward the negative source charge.



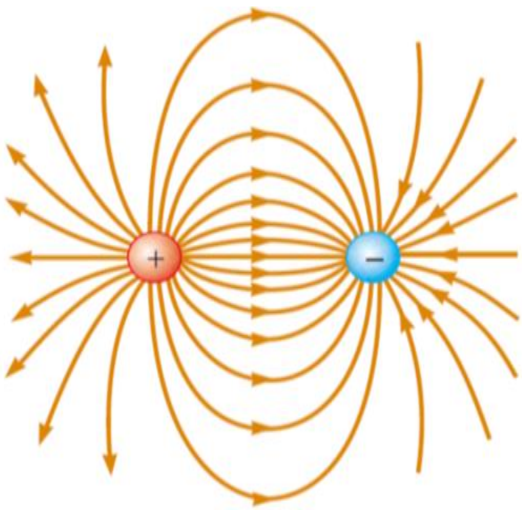
The electric field lines for two point charges (an electric dipole)

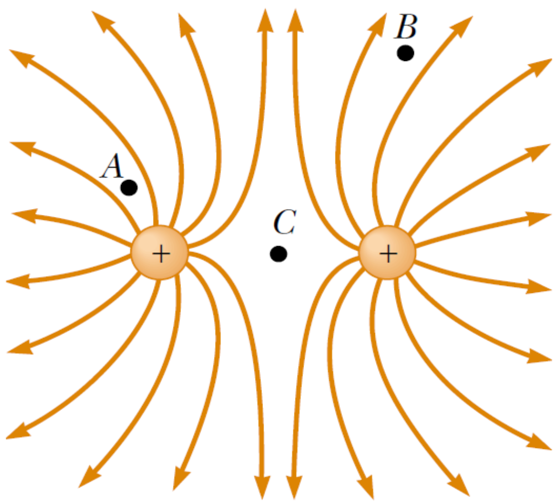
a) Unlike charges

- The charges are equal and opposite.
- The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge.

b) Like charges

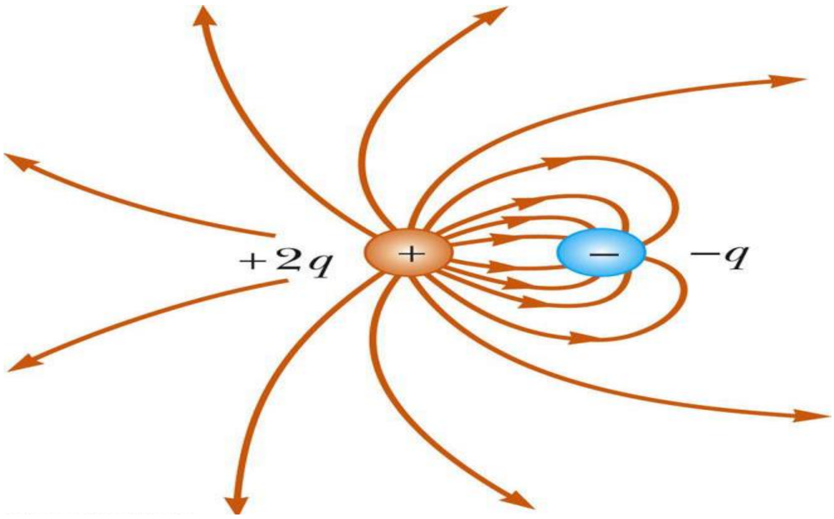
- The charges are equal and positive.
- The same number of lines leaves each charge since they are equal in magnitude.
- At a great distance, the field is approximately equal to that of a single charge of $2q$.





c) Unequal Charges

- The positive charge is twice the magnitude of the negative charge.
- Two lines leave the positive charge for each line that terminates on the negative charge.
- At a great distance, the field would be approximately the same as that due to a single charge of $+q$



Motion of Charged Particles in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field **E**, the electric force exerted on the charge is

$$\vec{F} = q\vec{E} = m\vec{a} \qquad \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

If **E** is uniform (that is, constant in magnitude and direction), then the acceleration is constant.

If the particle has a positive charge, its acceleration is in the direction of the electric field.

If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Electric Flux

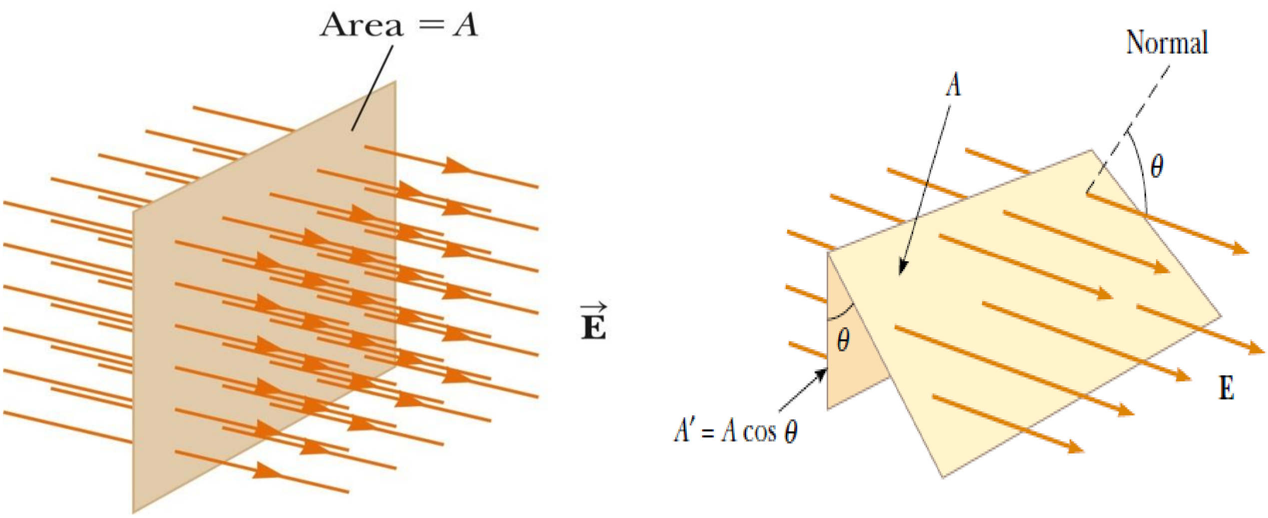
The field lines penetrate a surface of area A, whose plane is oriented perpendicular to the field; the number of lines per unit area (line density) is proportional to the magnitude of the electric field, as shown in (Figure 1 a). Therefore, the total number of lines penetrating the surface is proportional to the product **EA** is called the **electric flux** $\mathbf{\Phi}_E$

$$\Phi_E = EA \qquad (N.m^2/C)$$

If the electric field is uniform and makes an angle θ with the normal to a surface of area A, as shown in (Figure 1 b), then the electric flux through the surface is

$$\Phi_E = EA' = EA\cos\theta$$

The equations 1and 2 are showing the flux through a surface of fixed area (A) has a **maximum** value EA when the surface is **perpendicular** to the field, (when the normal to the surface is parallel to the field, that is, θ= 0°); the flux is **zero** when the surface is **parallel** to the field (when the normal to the surface is perpendicular to the field, that is, θ= 90°).



(a) **(b)**

Fig 1: The uniform electric field penetrating a plane of area A

a) perpendicular to the field. **b)** at an angle θ to the field.

Consider a general surface divided up into a large number of small elements, each of area ΔA (Figure 2 a). The electric field $\mathbf{E_i}$ at the location of this element makes an angle θ_i with the vector ΔA_i. The electric flux $\Delta\phi_E$ through this element is

$$\Phi_E = E_i \Delta A_i \cos\theta = \vec{E_i} \cdot \vec{\Delta A_i}$$

From the definition of the ***scalar product*** of two vectors

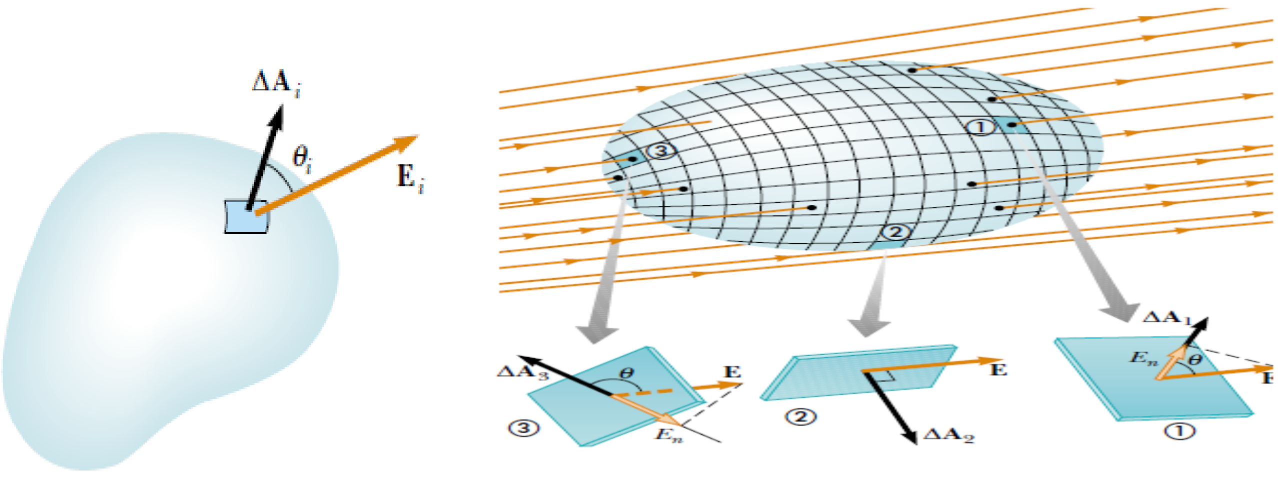
$$\vec{A}.\vec{B} = AB\cos\theta$$

In general, the electric flux through a surface is

The (Figure 2 b) shows that a net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number leaving the surface minus the number entering the surface. If more lines are leaving than entering, **the net flux is positive**. If more lines are entering than leaving, **the net flux is negative**; we can write the net flux ϕ_E through a closed surface as

$$\Phi_E = \oint \vec{E_i}.\vec{dA_i} = \oint E_n dA \qquad \text{.................. (5)}$$

where E_n represents the component of the electric field normal to the surface.



(a) **(b)**

Fig 2: a) The electric field makes an angle θ_i with a small element of surface area ΔA_i. **b)** The flux through an area element can be positive (element 1), zero (element 2) and negative (element 3).

Example 1: Flux Through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +1.00 μC at its center ?

Solution:

The magnitude of the electric field 1.00 m from this charge is given by Equation

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{1.00 \times 10^{-6}\ \mathrm{C}}{(1.00\ \mathrm{m})^2}$$
$$= 8.99 \times 10^3\ \mathrm{N/C}$$

The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6\ \mathrm{m^2}$) is thus

$$\Phi_E = EA = (8.99 \times 10^3\ \mathrm{N/C})(12.6\ \mathrm{m^2})$$
$$= 1.13 \times 10^5\ \mathrm{N \cdot m^2/C}$$

Example 2: Flux Through a Cube

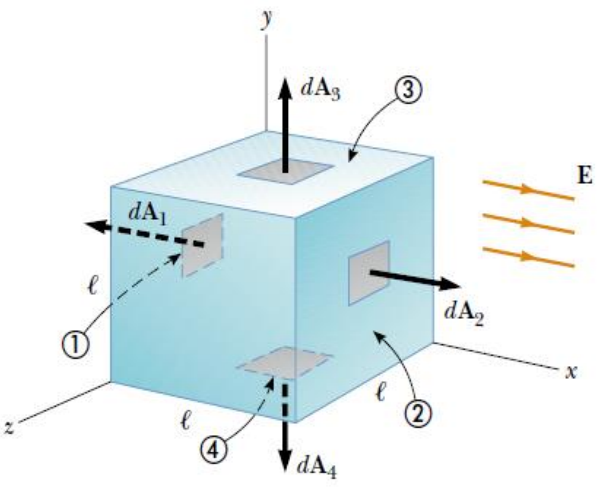
Consider a uniform electric field **E** oriented in the *x* direction. Find the net electric flux through the surface of a cube of edges ℓ, oriented as shown in Figure

Solution:

The net flux is the sum of the fluxes through all faces of the cube.

The net flux through faces 1 and 2 is

$$\Phi_E = \int_1 \mathbf{E} \cdot d\mathbf{A} + \int_2 \mathbf{E} \cdot d\mathbf{A}$$



For Face 1

E is constant and directed inward but $d\mathbf{A}_1$ is directed outward (θ=180°); thus, the flux through this face is

$$\int_1 \mathbf{E} \cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA$$

$$= -E\int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = \ell^2$.

For Face 2

E is constant and outward and in the same direction as $d\mathrm{A}_2$ ($\theta=0°$); hence, the flux through this face is

$$\int_2 \mathbf{E}\cdot d\mathbf{A} = \int_2 E(\cos 0°)\,dA = E\int_2 dA = +EA = E\ell^2$$

For Faces (3, 4, and the unnumbered ones):

$\mathbf{\Phi}_E = \mathbf{0}$ because **E** is perpendicular to d**A** on these faces.

Therefore, the net flux over all six faces is

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = 0$$

Gauss's Law

It describes a general relationship between the net electric flux through a closed surface (often called a Gaussian surface) and the charge enclosed by the surface.

Let us consider a positive point charge q located at the center of a sphere of radius r, as shown in (Figure 3 a). The net flux through the gaussian surface is

$$\Phi_E = \oint \vec{E}.\vec{dA} = \oint E dA = E\oint dA$$

$$\text{where} \quad E = K\frac{q}{r^2}\,, \quad K = \frac{1}{4\pi\varepsilon_0} \quad \text{and} \quad A = 4\pi r^2$$

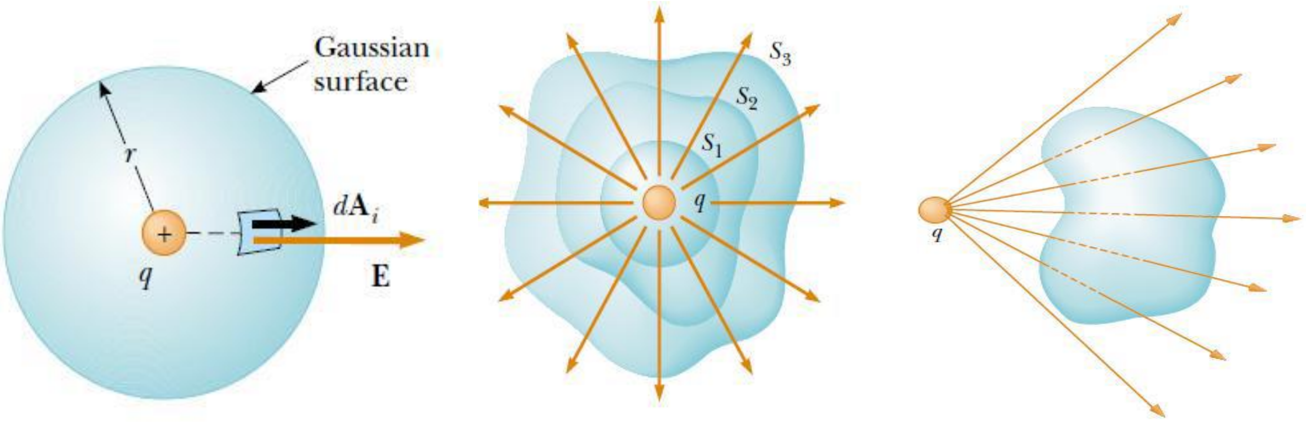
$$\Phi_E = K\frac{q}{r^2}\left(4\pi r^2\right)$$

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,4\pi r^2 \Rightarrow \Phi_E = \frac{q}{\varepsilon_0}$$

$$\Phi_E = \oint \vec{E}.\vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

where q_{in} represents the net charge inside the surface and **E** represents the electric field at any point on the surface.

Gauss's law says that the net electric flux through any closed gaussian surface is equal to the net charge q_{in} inside the surface divided by ε_0. The net electric flux is independent of the shape of that surface as shown in the following (Figure 3 b), When closed surfaces of various shapes surrounding a charge q, the net electric flux is the **same** through all surfaces. If a point charge located outside a closed surface, the number of lines entering the surface equals the number leaving the surface (see Figure 3 c), this means the net electric flux through a closed surface that surrounds no charge is zero.



(a) **(b)** **(c)**

Fig 3: a) spherical gaussian surface of radius *r* surrounding a point charge *q*. **b)** Closed surfaces of various shapes surrounding a charge *q*. **c)** A point charge located outside a closed surface.

As example: The net flux through surface S is q_1/ε_0 in the Figure 4.

The net flux through surface S′ is $(q_2 + q_3)/\varepsilon_0$, and the net flux through surface S″ is **zero**.

Charge q_4 does not contribute to the flux through any surface because it is outside all surfaces.

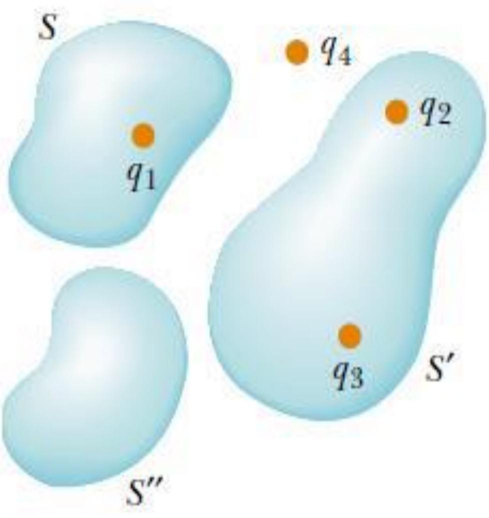


Fig 4: The net electric flux through any closed surface depends only on the charge *inside* that surface.

Note: The net electric flux through any closed surface depends only on the charge inside that surface.

The electric field due to **many charges** is the vector sum of the electric fields produced by the individual charges.

$$\oint \vec{E}.\vec{dA} = \oint \left(\vec{E_1} + \vec{E_2} + \ldots\ldots\right) . dA$$

Application of Gauss's Law to Various Charge Distributions

As mentioned earlier, Gauss's law is useful in determining electric fields when the charge distribution is characterized by a high degree of symmetry.

The goal in this type of calculation is to determine a surface that satisfies one or more of the following **conditions**:

1. The value of the electric field can be argued by symmetry to be constant over the surface.
2. The dot product of **E.dA** can be expressed as a simple product *E dA* because **E** and **dA** are parallel.
3. The dot product is zero because **E** and **dA** are perpendicular.
4. The field can be argued to be zero over the surface.

Example 3: The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge *q*.

Solution:

A single charge represents the simplest possible charge distribution, and we use this familiar case to show how to solve for the electric field with Gauss's law. We choose a spherical gaussian surface of radius *r* centered on the point charge, as shown in Figure. The electric field due to a positive point charge is directed radially outward by symmetry and is therefore normal to the surface at every point. Thus, as in condition (2), **E** is parallel to $d\mathbf{A}$ at each point. Therefore, $\mathbf{E} \cdot d\mathbf{A} = E\,dA$ and Gauss's law gives

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

By symmetry, E is constant everywhere on the surface, which satisfies condition (1), so it can be removed from the integral. Therefore,

$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

where we have used the fact that the surface area of a sphere is $4\pi r^2$. Now, we solve for the electric field:

$$E = \frac{q}{4\pi\epsilon_0 r^2} = k_e\frac{q}{r^2}$$

This is the familiar electric field due to a point charge that we developed from Coulomb's law.

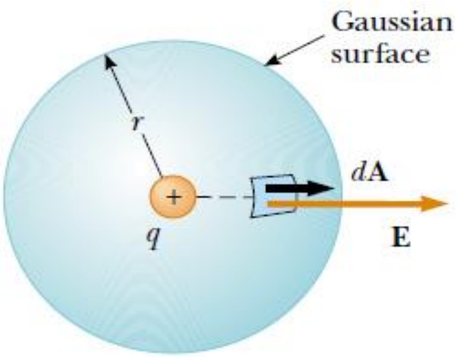


Fig 5: The point charge q is at the center of the spherical gaussian surface, and **E** is parallel to d**A** at every point on the surface.

Example 4: A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q (Fig 6).

(a) Calculate the magnitude of the electric field at a point outside the sphere.

Solution:

Because the charge distribution is spherically symmetric, we again select a spherical gaussian surface of radius r, concentric with the sphere, as shown in (Fig 6 a). For this choice, conditions (1) and (2) are satisfied, as they were for the point charge in Example 3. Following the line of reasoning given in Example 3, we find that

$$E = k_e\frac{Q}{r^2} \qquad (\text{for } r > a)$$

Note that this result is identical to the one we obtained for a point charge. Therefore, we conclude that, for a uniformly charged sphere, the field in the region external to the sphere is *equivalent* to that of a point charge located at the center of the sphere.

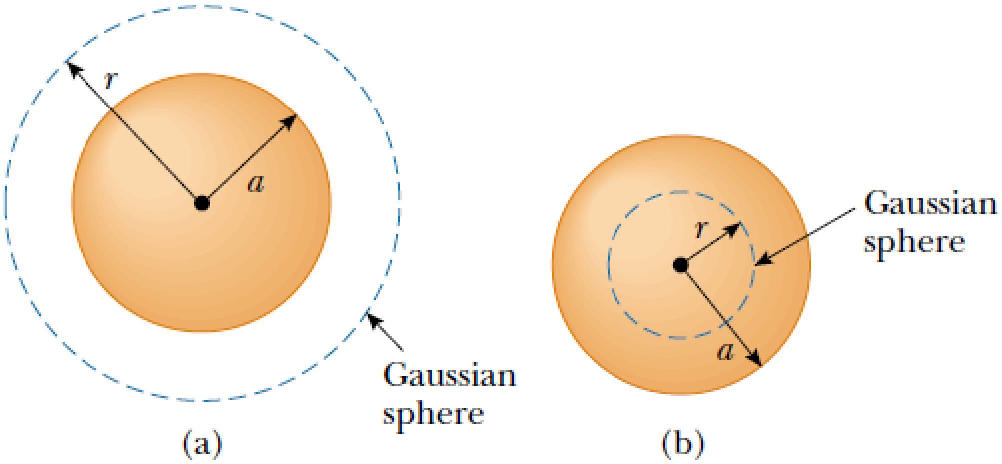


Fig 5: A uniformly charged insulating sphere of radius a and total charge Q. (a) The magnitude of the electric field at a point exterior to the sphere is $k_e\ Q\ /r^{\ 2}$. (b) The magnitude of the electric field inside the insulating sphere is due only to the charge *within* the gaussian sphere defined by the dashed circle and is $k_e\ Q\ r\ /a^3$.

(b) Find the magnitude of the electric field at a point inside the sphere.

Solution:

In this case we select a spherical gaussian surface having radius $r < a$, concentric with the insulated sphere (Fig. 6 b). Let us denote the volume of this smaller sphere by V'. To apply Gauss's law in this situation, it is important to recognize that the charge q_{in} within the Gaussian surface of volume V' is less than Q. To calculate q_{in} , we use the fact that $q_{in} = \rho V'$

$$q_{\text{in}} = \rho V' = \rho(\tfrac{4}{3}\pi r^3)$$

By symmetry, the magnitude of the electric field is constant everywhere on the spherical gaussian surface and is normal to the surface at each point—both conditions (1) and (2) are satisfied. Therefore, Gauss's law in the region $r < a$ gives

$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

Solving for E gives

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho\frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}r$$

Because

$$\rho = Q/\tfrac{4}{3}\pi a^3$$

by definition and since

$$k_e = 1/(4\pi\epsilon_0),$$

this expression for E can be written as

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \frac{k_e Q}{a^3}r \qquad (\text{for } r < a)$$

Note that this result for E differs from the one we obtained in part (a). It shows that $E \to 0$ as $r \to 0$. Therefore, the result eliminates the problem that would exist at $r = 0$ if E varied as $1/r^2$ inside the sphere as it does outside the sphere. That is, if $E \propto 1/r^2$ for $r < a$, the field would be infinite at $r = 0$, which is physically impossible. Note also that the expressions for parts (a) and (b) match when $r = a$.

A plot of E versus r is shown in Figure 6.

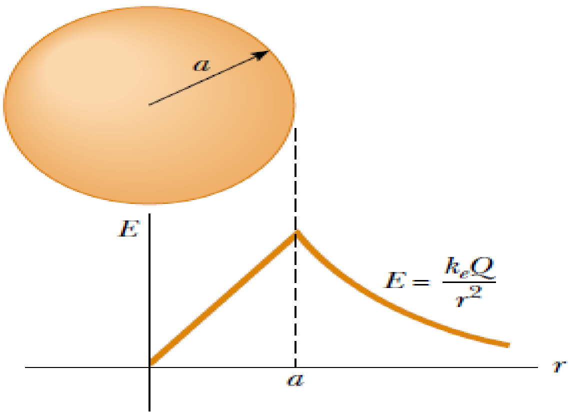


Fig 6: A plot of E versus r for a uniformly charged insulating sphere. The electric field inside the sphere ($r < a$) varies linearly with r. The field outside the sphere ($r > a$) is the same as that of a point charge Q located at $r = 0$.

Example 5: The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Fig. 7 a). Find the electric field at points (a) outside and (b) inside the shell.

Solution:

a) When gaussian surface of radius r > a as (Fig. 7 b), the charge inside this surface is Q. Therefore, the field at a point outside the shell is equivalent to that due to a point charge Q located at the center:

$$E = k_e \frac{Q}{r^2} \qquad (\text{for } r > a)$$

a) The electric field inside the spherical shell is zero (**E = 0**). This follows from Gauss's law applied to a spherical surface of radius $r < a$ concentric with the shell as (Fig. 7 c). Because of the spherical symmetry of the charge distribution and because the net charge inside the surface is zero—satisfaction of conditions (1) and (2) again—application of Gauss's law shows that $E = 0$ in the region $r < a$.

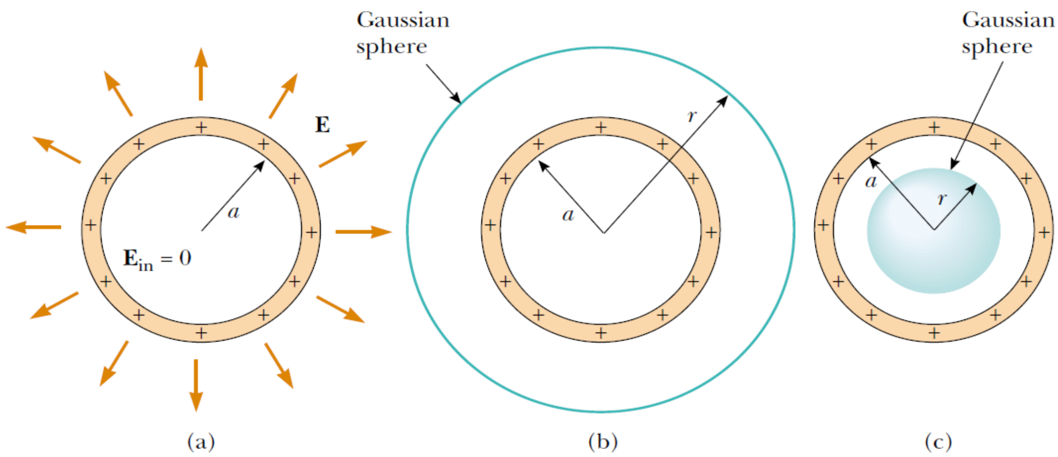


Fig. 7 (a) The electric field inside a uniformly charged spherical shell is zero. **(b)** Gaussian surface for r > a c) Gaussian surface for $r < a$.

Example 6: A Cylindrically Symmetric Charge Distribution

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ (Fig. 8 a).

Solution:

The symmetry of the charge distribution requires that **E** be perpendicular to the line charge and directed outward, (see following Fig 8 (a) and (b)).

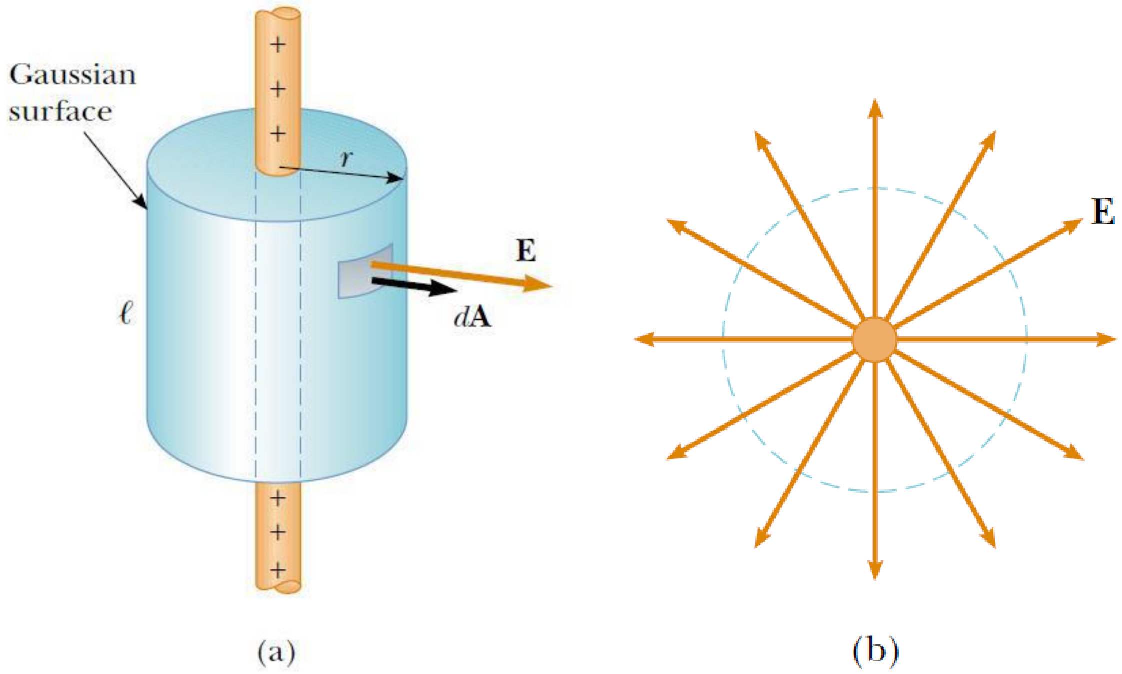


Fig. 8: (a) An infinite line of charge surrounded by a cylindrical gaussian surface concentric with the line. (b) An end view shows that the electric field at the cylindrical surface is constant in magnitude and perpendicular to the surface.

The total charge inside our gaussian surface is $\lambda\ell$. Applying Gauss's law and conditions (1) and (2), we find that for the curved surface

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E\oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda\ell}{\epsilon_0}$$

The area of the curved surface is $A = 2\pi r \ell$, therefore,

$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e\frac{\lambda}{r}$$

Electric Flux

The field lines penetrate a surface of area A, whose plane is oriented perpendicular to the field; the number of lines per unit area (line density) is proportional to the magnitude of the electric field, as shown in (Figure 1 a). Therefore, the total number of lines penetrating the surface is proportional to the product **EA** is called the **electric flux** $\mathbf{\Phi}_E$

$$\Phi_E = EA \qquad (\mathrm{N.m^2/C})$$

If the electric field is uniform and makes an angle θ with the normal to a surface of area A, as shown in (Figure 1 b), then the electric flux through the surface is

$$\Phi_E = EA' = EA\cos\theta$$

The equations 1and 2 are showing the flux through a surface of fixed area (A) has a **maximum** value EA when the surface is **perpendicular** to the field, (when the normal to the surface is parallel to the field, that is, θ= 0°); the flux is **zero** when the surface is **parallel** to the field (when the normal to the surface is perpendicular to the field, that is, θ= 90°).

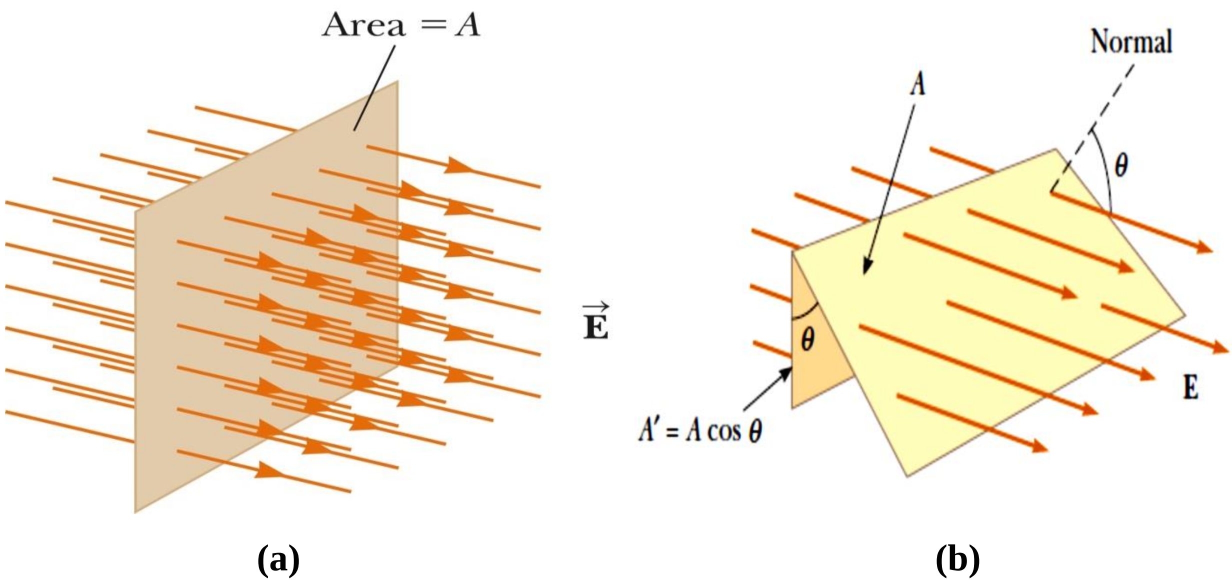


Fig 1: The uniform electric field penetrating a plane of area A

a) perpendicular to the field. **b)** at an angle θ to the field.

Consider a general surface divided up into a large number of small elements, each of area ΔA (Figure 2 a). The electric field $\mathbf{E_i}$ at the location of this element makes an angle θ_i with the vector ΔA_i. The electric flux $\Delta\phi_E$ through this element is

$$\Phi_E = E_i \Delta A_i \cos\theta = \vec{E_i} \cdot \vec{\Delta A_i}$$

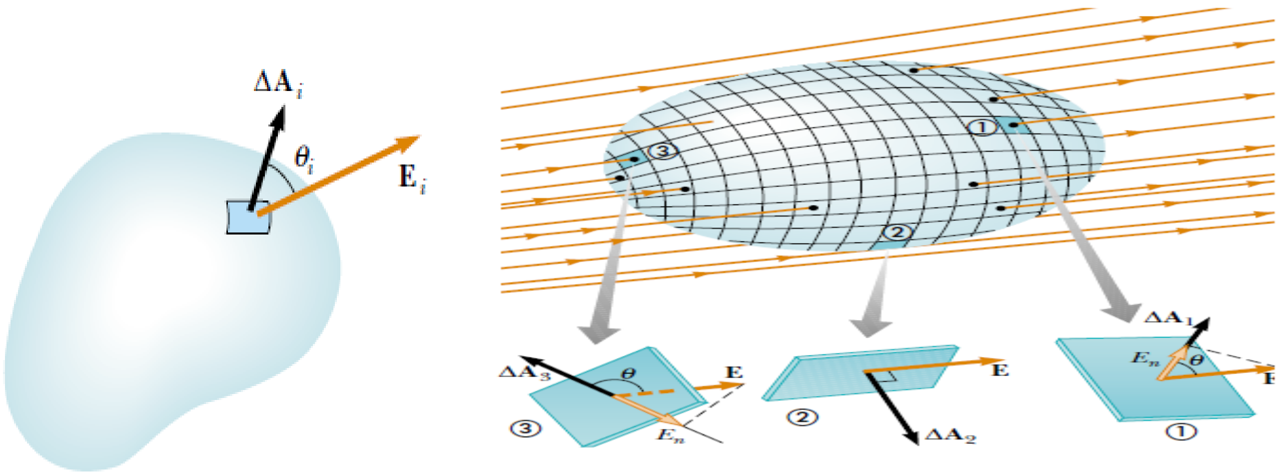
In general, the electric flux through a surface is

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \vec{E_i} \cdot \vec{\Delta A_i} = \int \vec{E_i} \cdot \vec{dA_i}$$

The (Figure 2 b) shows that a net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number leaving the surface minus the number entering the surface. If more lines are leaving than entering, **the net flux is positive**. If more lines are entering than leaving, **the net flux is negative**; we can write the net flux ϕ_E through a closed surface as

$$\Phi_E = \oint \vec{E_i} . \vec{dA_i} = \oint E_n dA \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where E_n represents the component of the electric field normal to the surface.



(a) **(b)**

Fig 2: a) The electric field makes an angle θ_i with a small element of surface area ΔA_i. **b)** The flux through an area element can be positive (element 1), zero (element 2) and negative (element 3).

Example 1: Flux Through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +1.00 μC at its center ?

Solution:

The magnitude of the electric field 1.00 m from this charge is given by Equation

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\, \text{N}\cdot\text{m}^2/\text{C}^2)\, \frac{1.00 \times 10^{-6}\,\text{C}}{(1.00\,\text{m})^2}$$

$$= 8.99 \times 10^3\,\text{N/C}$$

The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6$ m^2) is thus

$$\Phi_E = EA = (8.99 \times 10^3\,\text{N/C})(12.6\,\text{m}^2)$$

$$= 1.13 \times 10^5\,\text{N}\cdot\text{m}^2/\text{C}$$

Example 2: Flux Through a Cube

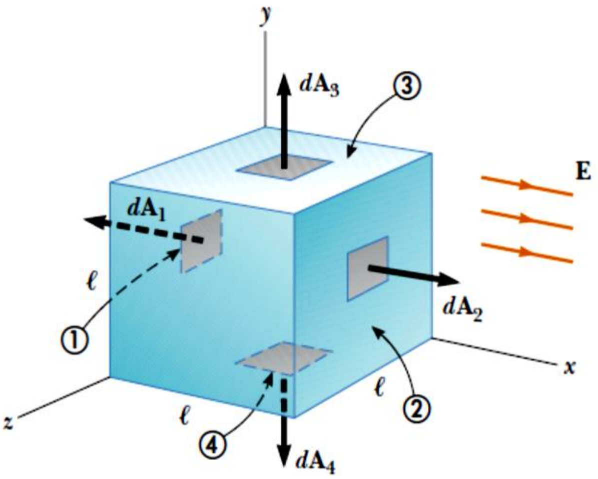
Consider a uniform electric field **E** oriented in the *x* direction. Find the net electric flux through the surface of a cube of edges ℓ, oriented as shown in Figure

Solution:

The net flux is the sum of the fluxes through all faces of the cube.

The net flux through faces 1 and 2 is

$$\Phi_E = \int_1 \mathbf{E}\cdot d\mathbf{A} + \int_2 \mathbf{E}\cdot d\mathbf{A}$$



For Face 1

E is constant and directed inward but $d\mathbf{A}_1$ is directed outward (θ=180°); thus, the flux through this face is

$$\int_1 \mathbf{E}\cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA$$

$$= -E\int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = \ell^2$.

For Face 2

E is constant and outward and in the same direction as $d\text{A}_2$ (θ=0°); hence, the flux through this face is

$$\int_2 \mathbf{E}\cdot d\mathbf{A} = \int_2 E(\cos 0°)\,dA = E\int_2 dA = +EA = E\ell^2$$

For Faces (3, 4, and the unnumbered ones):

$\mathbf{\Phi_E = 0}$ because **E** is perpendicular to d**A** on these faces.

Therefore, the net flux over all six faces is

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = 0$$

Gauss's Law

It describes a general relationship between the net electric flux through a closed surface (often called a Gaussian surface) and the charge enclosed by the surface.

Let us consider a positive point charge q located at the center of a sphere of radius r, as shown in (Figure 3 a). The net flux through the gaussian surface is

$$\Phi_E = \oint \vec{E}.\vec{dA} = \oint E dA = E\oint dA$$

where $E = K\frac{q}{r^2}$, $K = \frac{1}{4\pi\varepsilon_0}$ and $A = 4\pi r^2$

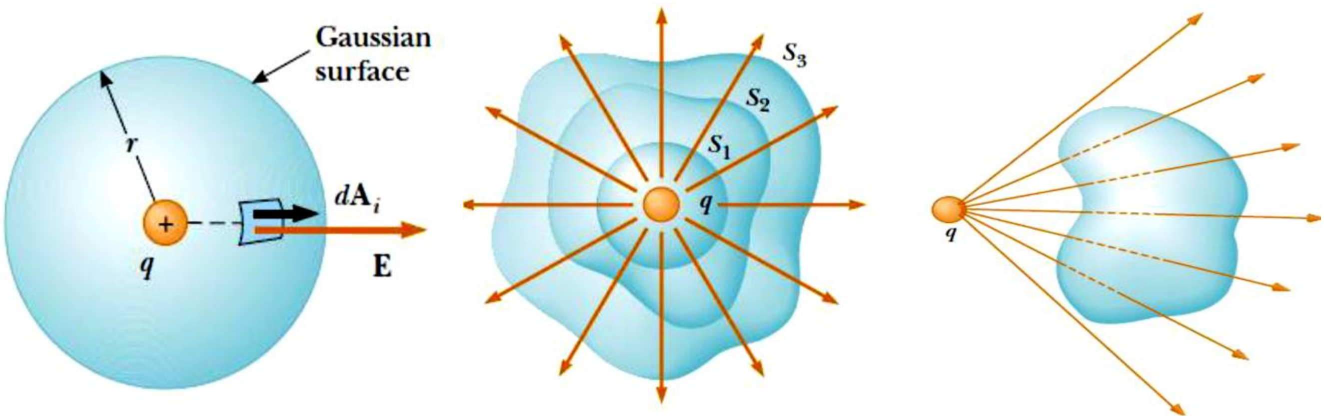
$$\Phi_E = K\frac{q}{r^2}\left(4\pi r^2\right)$$

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\;4\pi r^2 \Rightarrow \Phi_E = \frac{q}{\varepsilon_0}$$

$$\Phi_E = \oint \vec{E}.\vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

where q_{in} represents the net charge inside the surface and **E** represents the electric field at any point on the surface.

Gauss's law says that the net electric flux through any closed gaussian surface is equal to the net charge q_{in} inside the surface divided by ε_0. The net electric flux is independent of the shape of that surface as shown in the following (Figure 3 b), When closed surfaces of various shapes surrounding a charge q, the net electric flux is the **same** through all surfaces. If a point charge located outside a closed surface, the number of lines entering the surface equals the number leaving the surface (see Figure 3 c), this means the net electric flux through a closed surface that surrounds no charge is zero.



(a) **(b)** **(c)**

Fig 3: a) spherical gaussian surface of radius r surrounding a point charge q. **b)** Closed surfaces of various shapes surrounding a charge q. **c)** A point charge located outside a closed surface.

As example: The net flux through surface S is q_1/ε_0 in the Figure 4.

The net flux through surface S′ is $(q_2 + q_3)/\varepsilon_0$, and the net flux through surface S″ is **zero**.

Charge q_4 does not contribute to the flux through any surface because it is outside all surfaces.

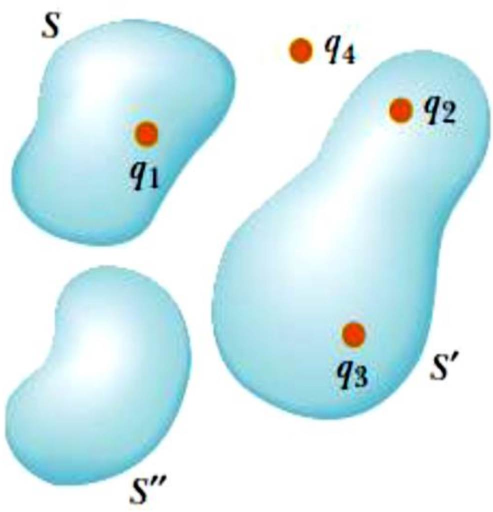


Fig 4: The net electric flux through any closed surface depends only on the charge *inside* that surface.

Note: The net electric flux through any closed surface depends only on the charge inside that surface.

The electric field due to **many charges** is the vector sum of the electric fields produced by the individual charges.

$$\oint \vec{E}.\vec{dA} = \oint \left(\vec{E_1} + \vec{E_2} + \ldots\ldots\right) . dA$$

Application of Gauss's Law

As mentioned earlier, Gauss's law is useful in determining electric fields when the charge distribution is characterized by a high degree of symmetry.

The goal in this type of calculation is to determine a surface that satisfies one or more of the following **conditions**:

1. The value of the electric field can be argued by symmetry to be constant over the surface.
2. The dot product of **E.dA** can be expressed as a simple product $E\ dA$ because **E** and **dA** are parallel.
3. The dot product is zero because **E** and **dA** are perpendicular.
4. The field can be argued to be zero over the surface.

Example 3: The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.

Solution:

A single charge represents the simplest possible charge distribution, and we use this familiar case to show how to solve for the electric field with Gauss's law. We choose a spherical gaussian surface of radius r centered on the point charge, as shown in (Figure 5).

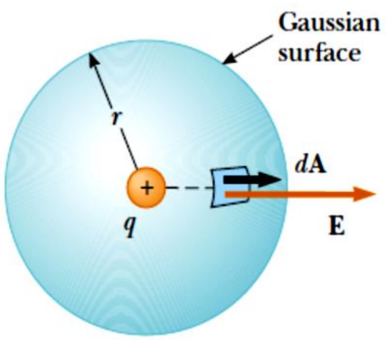


Fig 5: The point charge q is at the center of the spherical gaussian surface, and **E** is parallel to d**A** at every point on the surface.

The electric field due to a positive point charge is directed radially outward by symmetry and is therefore normal to the surface at every point. Thus, as in **condition (2)**, **E** is parallel to $d\mathbf{A}$ at each point. Therefore, $\mathbf{E} \cdot d\mathbf{A} = E\, dA$ and Gauss's law gives:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\, dA = \frac{q}{\epsilon_0}$$

By symmetry, E is constant everywhere on the surface, which satisfies **condition (1)**, so it can be removed from the integral. Therefore,

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

where we have used the surface area of a sphere is $4\pi r^2$.

Now, we solve for the electric field:

$$E = \frac{q}{4\pi\epsilon_0 r^2} = k_e \frac{q}{r^2}$$

This is the electric field due to a point charge that we developed from Coulomb's law.

Example 4: A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q (Figure 6).

(a) Calculate the magnitude of the electric field at a point outside the sphere.

Solution:

Because the charge distribution is spherically symmetric, we again select a spherical gaussian surface of radius r, concentric with the sphere, as shown in (Figure 6 a). For this choice, **conditions (1) and (2)** are satisfied, as they were for the point charge in Example 3. We find that:

$$E = k_e \frac{Q}{r^2} \qquad (\text{for } r > a)$$

Note that this result is identical to the one we obtained for a point charge.

Therefore, we conclude that, for a uniformly charged sphere, the field in the region external to the sphere is *equivalent* to that of a point charge located at the center of the sphere.

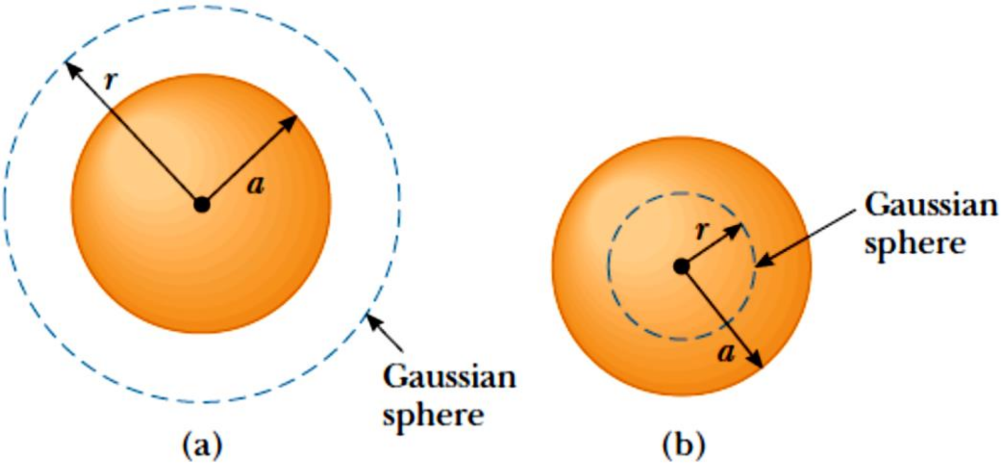


Fig 6: A uniformly charged insulating sphere of radius a and total charge Q. **(a)** The magnitude of the electric field at a point exterior to the sphere is $k_e\, Q\, /r^{2}$. **(b)** The magnitude of the electric field inside the insulating sphere is due only to the charge *within* the gaussian sphere defined by the dashed circle and is $k_e\, Q\, r\, /a^3$.

(b) Find the magnitude of the electric field at a point inside the sphere.

Solution:

In this case we select a spherical gaussian surface having radius $r < a$, concentric with the insulated sphere (Figure 6 b).

Let us denote the volume of this smaller sphere by V'. To apply Gauss's law in this situation, it is important to recognize that the charge q_{in} within the Gaussian surface of volume V' is **less than Q**.

To calculate q_{in} , we use the fact that $q_{in} = \rho V'$

$$q_{\text{in}} = \rho V' = \rho(\tfrac{4}{3}\pi r^3)$$

By symmetry, the magnitude of the electric field is constant everywhere on the spherical gaussian surface and is normal to the surface at each point, both **conditions (1) and (2)** are satisfied.

Therefore, Gauss's law in the region $r < a$ gives

$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

Solving for E gives

$$E = \frac{q_{in}}{4\pi\epsilon_0 r^2} = \frac{\rho\frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}r$$

Because

$$\rho = Q/\tfrac{4}{3}\pi a^3$$
$$k_e = 1/(4\pi\epsilon_0),$$

this expression for E can be written as

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \frac{k_e Q}{a^3}r \qquad (\text{for } r < a)$$

Note that this result for E differs from the one we obtained in part (a).

Example 5: The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Figure 7 a). Find the electric field at points (a) outside and (b) inside the shell.

Solution:

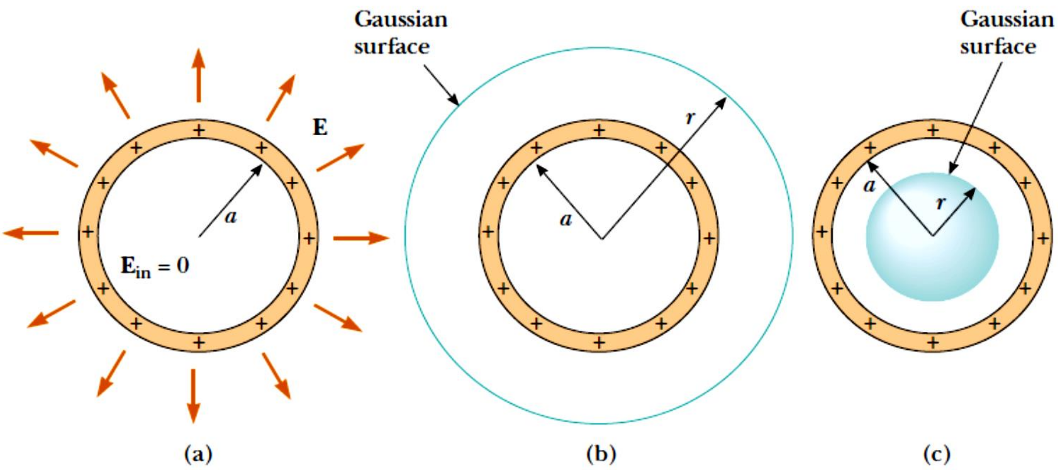


Fig 7: (a) The electric field inside a uniformly charged spherical shell is zero. The field outside is the same as that due to a point charge Q located at the center of the shell. **(b)** Gaussian surface for $r > a$. **(c)** Gaussian surface for $r < a$.

a) If we construct a spherical gaussian surface of radius $r > a$ concentric with the shell (Figure 7 b), the charge inside this surface is Q.

Therefore, the field at a point outside the shell is equivalent to that due to a point charge Q located at the center:

$$E = k_e \frac{Q}{r^2} \qquad \text{(for } r > a\text{)}$$

b) The electric field inside the spherical shell is zero (**E = 0**).

This follows from Gauss's law applied to a spherical surface of radius $r < a$ concentric with the shell as (Figure 7 c).

Because of the spherical symmetry of the charge distribution and because the net charge inside the surface is zero, satisfaction of **conditions (1) and (2)**, again application of Gauss's law shows that $\boldsymbol{E = 0}$ in the region $\boldsymbol{r < a}$.

Example 6: A Cylindrically Symmetric Charge Distribution

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ (Figure 8 a).

Solution:

The symmetry of the charge distribution requires that **E** be **perpendicular** to the line charge and directed outward, as shown in (Figure 8 (a) and (b)).

To reflect the symmetry of the charge distribution, we select a cylindrical gaussian surface of radius r and length ℓ that is coaxial with the line charge.

For the curved part of this surface, **E** is constant in magnitude and perpendicular to the surface at each point, satisfaction of **conditions (1) and (2)**.

Furthermore، the flux through the ends of the gaussian cylinder is zero because E is parallel to these surfaces, the first application we have seen of **condition (3)**.

The total charge inside our gaussian surface is $\lambda\ell$. Applying Gauss's law and **conditions (1) and (2)**, we find that for the curved surface

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

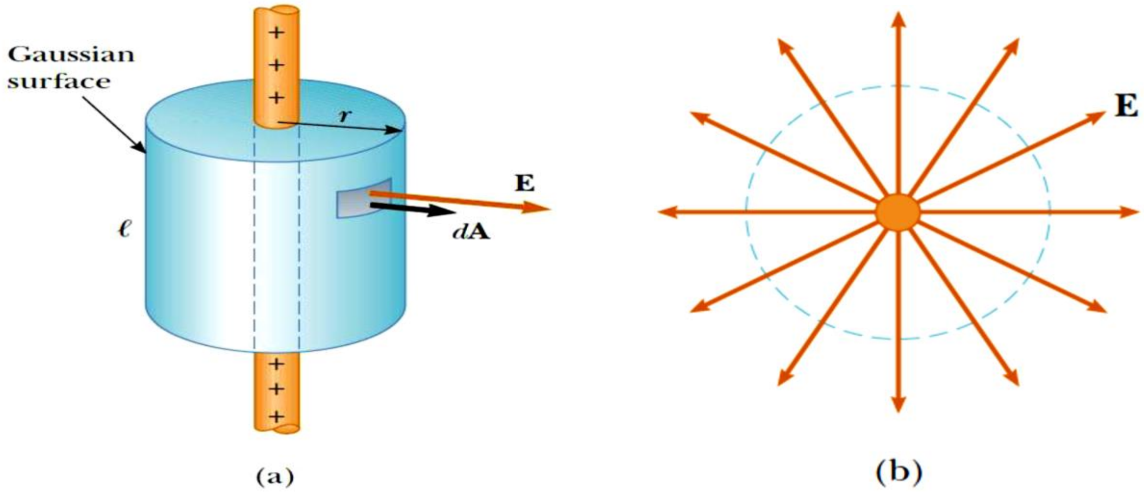


Fig 8: (a) An infinite line of charge surrounded by a cylindrical gaussian surface concentric with the line. **(b)** An end view shows that the electric field at the cylindrical surface is **constant in magnitude** and **perpendicular** to the surface.

The area of the curved surface is $A = 2\pi\, r\, \ell$, therefore,

$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e\frac{\lambda}{r}$$

Conductors in Electrostatic Equilibrium

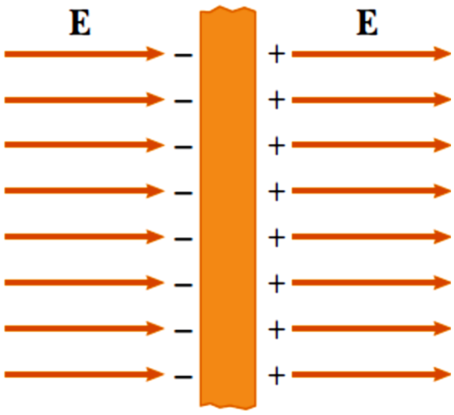
A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material.

When there is no net motion of charge within a conductor, the conductor is in **electrostatic equilibrium**. A conductor in electrostatic equilibrium has the following properties:

1. The electric field is **zero** everywhere inside the conductor.
2. If an isolated conductor carries a charge, the charge resides on its surface.
3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ / ε_0 , where σ is the surface charge density at that point.

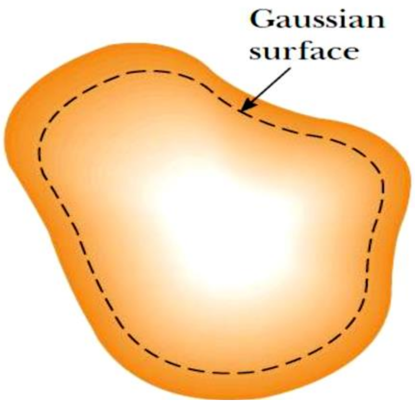
4. On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.

Consider a conducting slab in an external field.



- If the field inside the conductor were **not zero**, free electrons in the conductor would experience an electrical force.
- These electrons would accelerate.
- These electrons would not be in equilibrium.
- Therefore, there cannot be a field inside the conductor.

Choose a gaussian surface inside but close to the actual surface.



- The electric field inside is zero.
- There is no net flux through the gaussian surface.
- Because the gaussian surface can be as close to the actual surface as desired, there can be no charge inside the surface.

Applying Gauss's law to this surface, we obtain:

$$\Phi_E = \oint E\, dA = EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

where we have used the fact that $q_{in} = \sigma A$. Solving for E gives

$$E = \frac{\sigma}{\epsilon_0}$$

Electricity and Magnetism

Dr. Shurooq Saad Mahmood

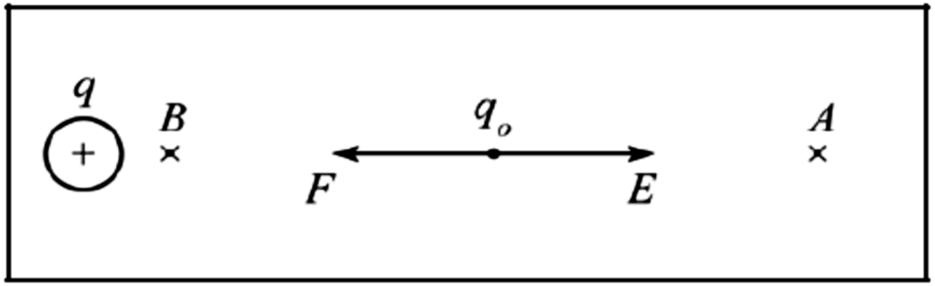
Electric Potential

1. Potential Difference and Electric Potential

When a **test charge q_0** is placed in an **electric field E** created by some other charged object, the **electric force** acting on the **test charge** is **q_0 E**.

$$\vec{\mathbf{F}} = q_o\vec{\mathbf{E}}$$

The force **q_0 E** is **conservative**, because the force between charges described by Coulomb's law is conservative. If the test charge is moved in the field by some external agent from point **A** to point **B** by a displacement **ds**, the **work** done by the **electric field** on the charge is **equal** to the **negative** of the **work** done by the **external agent** causing the **displacement**.



1. Potential Difference and Electric Potential

For an infinitesimal displacement **ds**, the **work** done by the **electric field** on the charge is:

$$W = \vec{F} \cdot \vec{ds} \Rightarrow W = q_0 \vec{E} \cdot \vec{ds}$$

As this amount of **work** is done by the **electric field**, the **potential energy** of the charge field system is **decreased** by an amount:

$$dU = -q_0 \vec{E} . \vec{ds}$$

The **change in potential energy** of the system is:

$$\Delta U = U_B - U_A$$

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \quad \text{.................. (1)}$$

1. Potential Difference and Electric Potential

The **potential energy per unit charge** U/q_0 is independent of the value of q_0 and has a value at every point in an electric field. This quantity U/q_0 is called the **electric potential V**.

Thus, the **electric potential** at any point in an **electric field** is

$$V = \frac{U}{q_0} \qquad \text{.................. (2)}$$

Note: The fact that **potential energy U** is a **scalar quantity** means that **electric potential V** also is a **scalar quantity**.

1. Potential Difference and Electric Potential

The **potential difference** $\Delta V = V_B - V_A$ between any two points **A** and **B** in an **electric field** is defined as the change in potential energy of the system divided by the test charge q_0 :

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \text{………………..} (3)$$

The **SI** unit of both **electric potential** and **potential difference** is joules J per coulomb C, which is defined as a volt (V):

1 V = 1 J/C

2. Potential Differences in a Uniform Electric Field

Let us calculate the **potential difference** between two points **A** and **B** separated by a distance ***d***, where ***d*** is parallel to the field lines. Equation 3 gives:

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E \cos 0° \, ds = -\int_A^B E \, ds$$

Because **E** is constant, we can remove it from the integral sign; this gives:

$$\Delta V = -E \int_A^B ds = -Ed \qquad \text{.................. (4)}$$

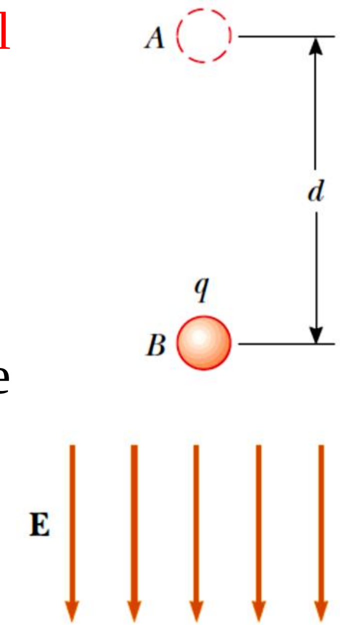


Fig. 1

The **negative** sign indicates that the **electric potential** at point **B** is lower than at point **A**; that is, $V_B < V_A$.

Electric field lines always point in the **direction of decreasing electric potential**, as shown in Figure 1.

2. Potential Differences in a Uniform Electric Field

When the electric field **E** is directed downward as shown in Figure 1, a point **B** is at a **lower electric potential** than point **A**. When a positive test charge moves from point **A** to point **B**, its **loses electric potential energy**.

Now suppose that a **test charge** q_0 moves from **A** to **B**. We can calculate the **change in its potential energy** from Equations 3 and 4:

$$\Delta U = q_0 \Delta V = -q_0 E d \qquad \text{……………….. (5)}$$

From this result, if q_0 is **positive**, then **ΔU** is **negative**. We conclude that a **positive charge** **loses** **electric potential energy** when it moves in the **direction of the electric field**.

While q_0 is **negative**, then **ΔU** is **positive** and the situation is reversed: A **negative charge** **gains** **electric potential energy** when it moves in the **direction of the electric field**.

2. Potential Differences in a Uniform Electric Field

Now consider the more general case of a **charged particle** that moves between **A** and **B** in a **uniform electric field** such that the **vector s** is **not parallel** to the **field lines**, as shown in Figure 2.

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\mathbf{E} \cdot \int_A^B d\mathbf{s} = -\mathbf{E} \cdot \mathbf{s}$$

The **change in potential energy** of the charge is:

$$\Delta U = q_0 \, \Delta V = -q_0 \, \mathbf{E} \cdot \mathbf{s}$$

The dot product for $\mathbf{s}_{A \to C}$, where $\theta = 0$ Therefore, $V_B = V_C$.

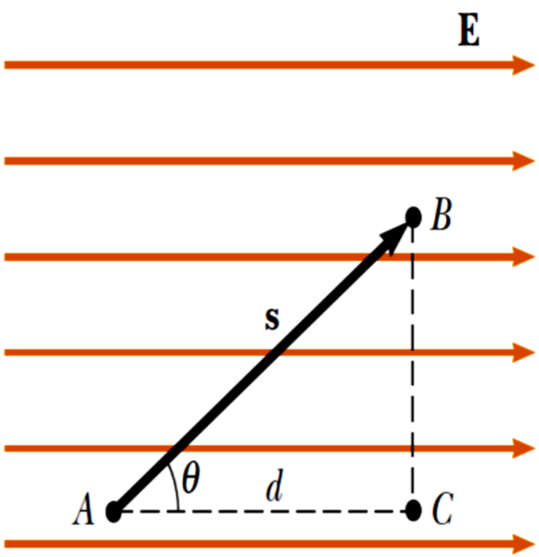


Fig. 2: A uniform electric field directed along the positive x axis.

The name **equipotential surface** is given to any surface consisting of a **continuous distribution of points** having the **same electric potential**.

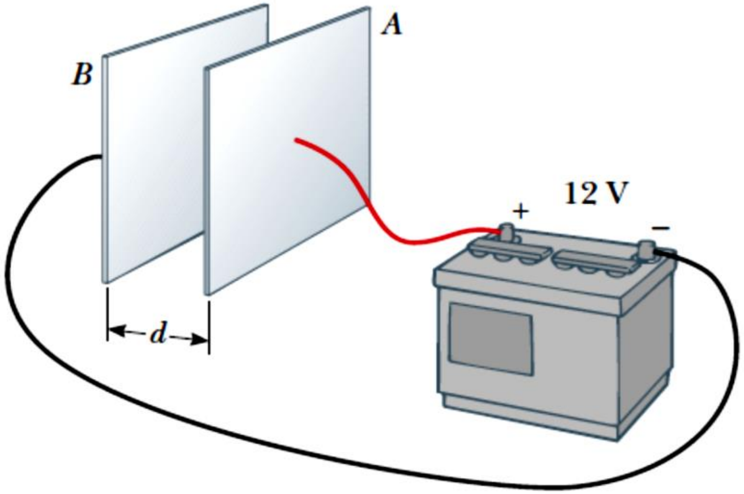
2. Potential Differences in a Uniform Electric Field

Example 1: A battery produces a specified potential difference **ΔV** between conductors attached to the battery terminals. A 12 V battery is connected between two parallel plates. The separation between the plates is **d** = 0.3 cm. Find the magnitude of the electric field between the plates.

Solution:

$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30 \times 10^{-2}\ \text{m}}$$

$$= 4.0 \times 10^{3}\ \text{V/m}$$



2. Potential Differences in a Uniform Electric Field

Example 2: A proton is released from rest in a uniform electric field that has a magnitude of 8×10^{4} V/m and is directed along the positive x axis as shown in Figure 3. The proton undergoes a displacement of 0.5 m in the direction of E. **(A)** Find the change in electric potential between points **A** and **B**.

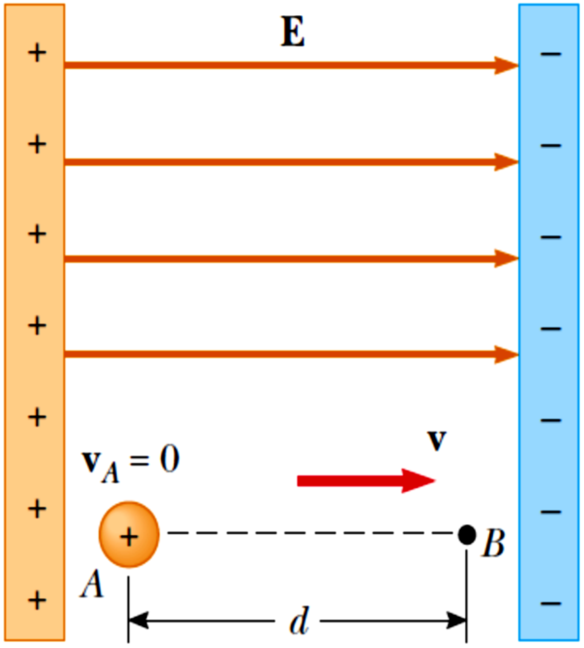


Fig. 3

Solution:

Because the proton (carries a **positive charge**) moves in the **direction of the field**, we expect it to move to a position of **lower electric potential**.

From Equation 4, we have

$$\Delta V = -Ed = -(8.0\times10^{4}\ \mathrm{V/m})(0.50\ \mathrm{m})$$

$$= -4.0\times10^{4}\ \mathrm{V}$$

2. Potential Differences in a Uniform Electric Field

(B) Find the change in potential energy of the proton for this displacement.

Solution:

$$\Delta U = q_0 \, \Delta V = e \, \Delta V$$
$$= (1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^{4} \text{ V})$$
$$= -6.4 \times 10^{-15} \text{ J}$$

The **negative sign** means the **potential energy** of the proton **decreases** as the proton moves in the **direction of the electric field**.

As the proton **accelerates** in the **direction of the field**, it gains kinetic energy and at the same time **loses** **electric potential energy**.

Electricity and Magnetism

Dr. Shurooq Saad Mahmood

Electric Potential (2)

3. Electric Potential and Potential Energy Due to Point Charges

Consider a **positive point charge *q*** produces an **electric field** that is directed radially outward from the charge (see Figure 4). To find the **electric potential** at a point located a distance ***r*** from the charge, we begin with the general expression for **potential difference**:

$$V_B - V_A = -\int_A^B \vec{E}.\vec{ds}$$

At any point in space, the **electric field** due to the point charge is:

$$\vec{E} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}}$$

Where ***r^*** is a **unit vector** directed from the charge toward the point.

$$E.ds = k_e \frac{q}{r^2}\hat{r}.\overrightarrow{ds} = k_e \frac{q}{r^2} ds\cos\theta$$

where θ is the angle between ***r^*** and ***ds***.

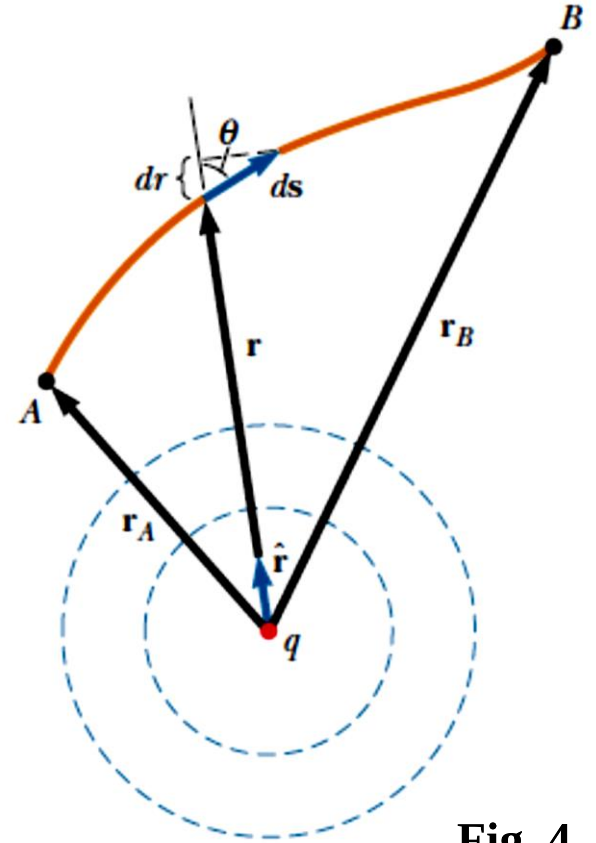


Fig. 4

3. Electric Potential and Potential Energy Due to Point Charges

$$= k_e \frac{q}{r^2} dr$$

$$V_B - V_A = -\int_A^B E\, dr = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2}$$

$$= \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

It is customary to choose the reference of **electric potential** for a point charge to be zero (**V = 0**) at $r_A = \infty$.

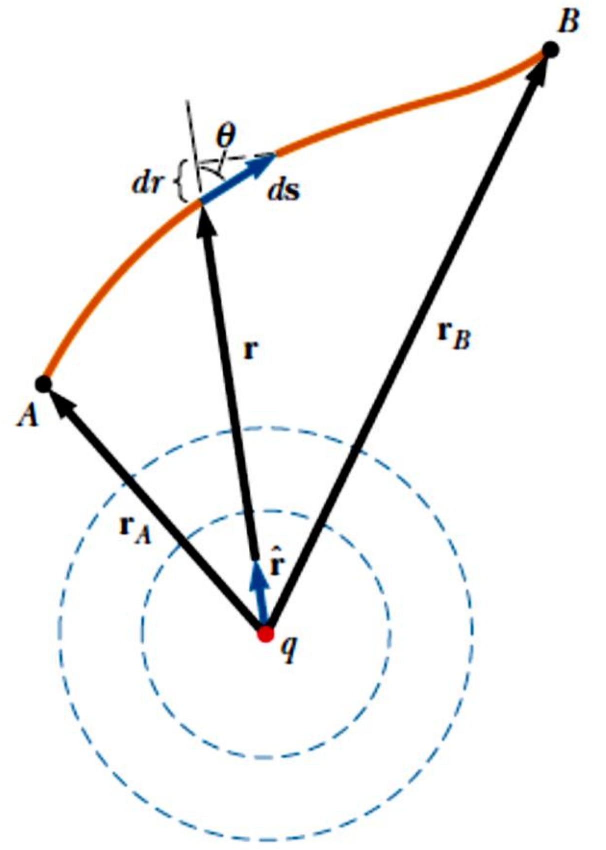


Fig. 4

3. Electric Potential and Potential Energy Due to Point Charges

The **electric potential** created by a point charge at any distance ***r*** from the charge is:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

The **total electric potential** at some point ***P*** due to several point charges is:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} \quad , \quad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$V = V_1 + V_2 + \ldots\ldots\ldots\ldots\ldots\ldots + V_n = \sum_n V_n$$

$$\therefore V = \frac{1}{4\pi\varepsilon_0}\sum_n \frac{q_n}{r_n}$$

3. Electric Potential and Potential Energy Due to Point Charges

The **potential energy U** when the two particles are separated by a distance $\boldsymbol{r_{12}}$ (see Figure 5)

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

Note that if the charges are of the **same sign**, **U** is **positive**. This is consistent with the fact that **positive work** must be done by an **external agent** on the system to bring the two charges near one another.

If the charges are of **opposite sign**, **U** is **negative**; this means that **negative work** is done by an external agent against on theirs.

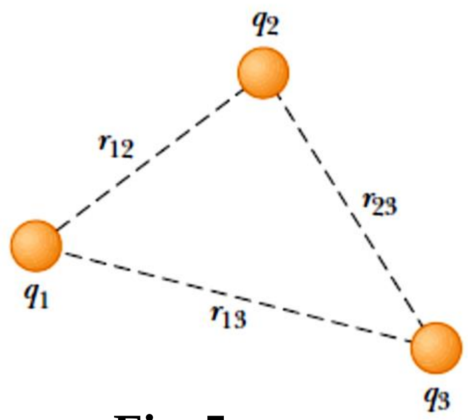


Fig. 5

If the system consists of more than two charged particles as shown in the Figure 5, then total potential energy of the system U is:

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

3. Electric Potential and Potential Energy Due to Point Charges

Example 3: A charge $\boldsymbol{q_1}$ **= 2.00 μC** is located at the origin, and a charge $\boldsymbol{q_2}$ **= - 6.00 μC** is located at (0, 3.00) m, as shown in Figure 6a. **(a)** Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

Solution:

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{2.00 \times 10^{-6}\text{ C}}{4.00\text{ m}} + \frac{-6.00 \times 10^{-6}\text{ C}}{5.00\text{ m}}\right)$$

$$= \boxed{-6.29 \times 10^3\text{ V}}$$

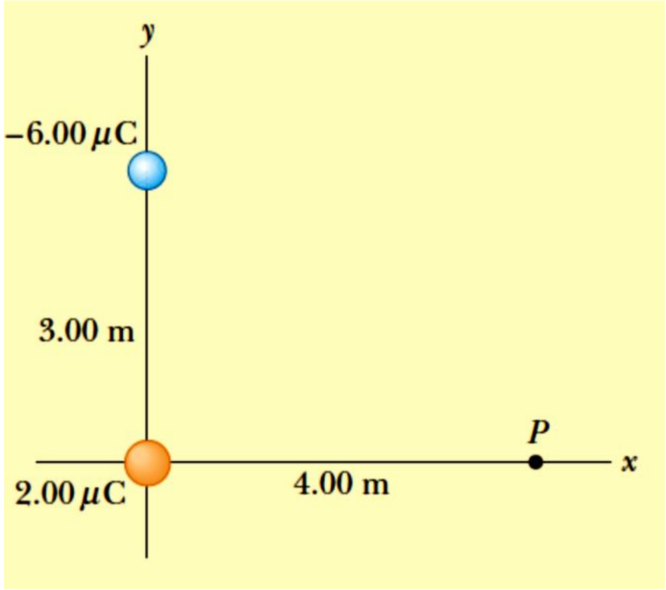


Fig. 6 (a)

3. Electric Potential and Potential Energy Due to Point Charges

(**b**) Find the change in potential energy of the system of two charges plus a charge $\boldsymbol{q_3} = \mathbf{3.00\ \mu C}$ as the latter charge moves from infinity to point *P* (Figure 6b).

Solution:

$$\Delta U = U_f - U_i$$

When the charge is at infinity, $U_i = 0$, and when the charge is at *P*, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^{3}\ \text{V})$$

$$= -18.9 \times 10^{-3}\ \text{J}$$

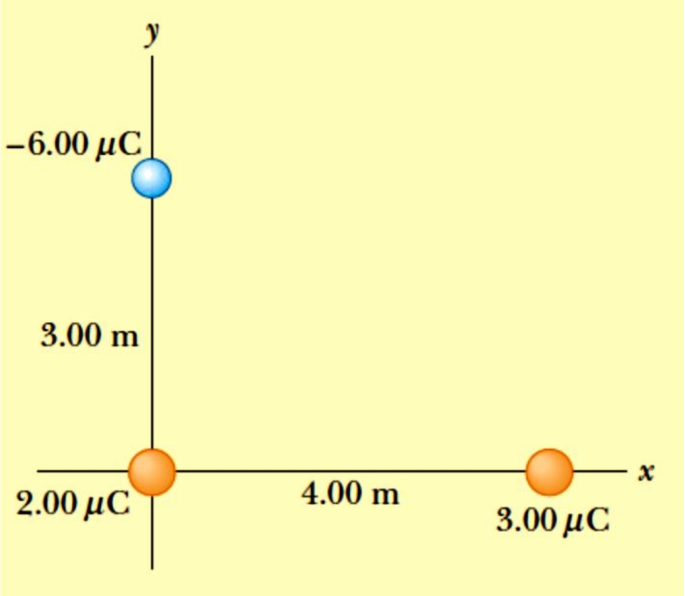


Fig. 6 (b)

3. Electric Potential and Potential Energy Due to Point Charges

(**c**) Find the potential energy of the system of three charges (Fig. 6b).

Solution:

$$U = K\left(\frac{q_1\,q_2}{r_{12}} + \frac{q_1\,q_3}{r_{13}} + \frac{q_2\,q_3}{r_{23}}\right)$$

$$U = \left(9\times10^{9}\right)\left(\frac{\left(2\times10^{-6}\right)\left(6\times10^{-6}\right)}{3} + \frac{\left(2\times10^{-6}\right)\left(3\times10^{-6}\right)}{4} + \frac{\left(-6\times10^{-6}\right)\left(3\times10^{-6}\right)}{5}\right)$$

$$U \approx -5.5\times10^{-2}\,\text{J}$$

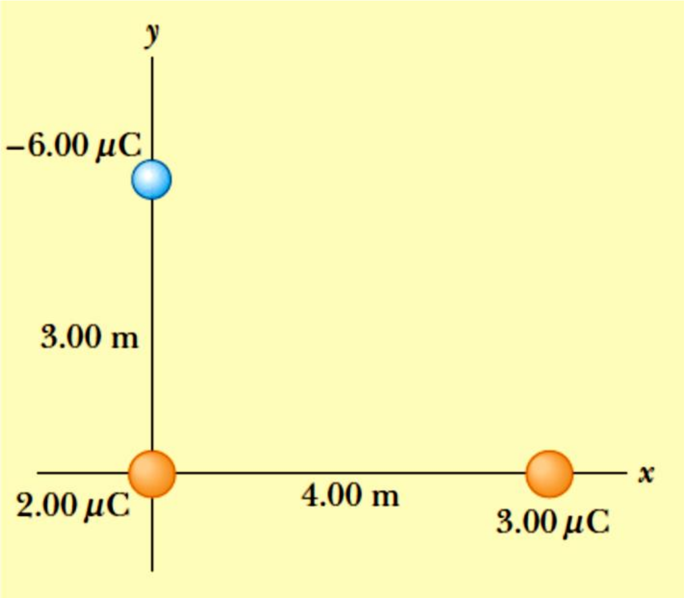


Fig. 6 (b)

4. Obtaining the value of the electric field From the electric potential

We can express the **potential difference** $\boldsymbol{dV}$ between two points a distance $\boldsymbol{ds}$ apart as:

$$dV = -\mathbf{E} \cdot d\mathbf{s}$$

If the electric field has only one component E_x , then $\mathbf{E.ds = E_x\, dx}$. Therefore, above Equation becomes $\boldsymbol{dV = - E_x\, dx}$, or

$$E_x = -\frac{dV}{dx}$$

That is, the x component of the electric field is equal to the **negative** of the **derivative** of the electric potential with respect to x. Similar statements can be made about the y and z components.

If the **charge distribution** creating an **electric field** has spherical symmetry such that the **volume charge density** depends only on the **radial distance r**, then the **electric field is radial**.

$$E_r = -\frac{dV}{dr}$$

4. Obtaining the value of the electric field From the electric potential

When a **test charge** undergoes a **displacement *ds*** along an **equipotential surface**, then ***dV = 0*** because the **potential is constant along an equipotential surface**.

From Equation $dV = -\mathbf{E} \cdot d\mathbf{s}$, we see that **dV = E.ds = 0**; thus, **E** must be **perpendicular** to the **displacement** along the equipotential surface. This shows that the **equipotential surfaces** must always be perpendicular to the **electric field lines** passing through them. as shown in the (Figure 7).

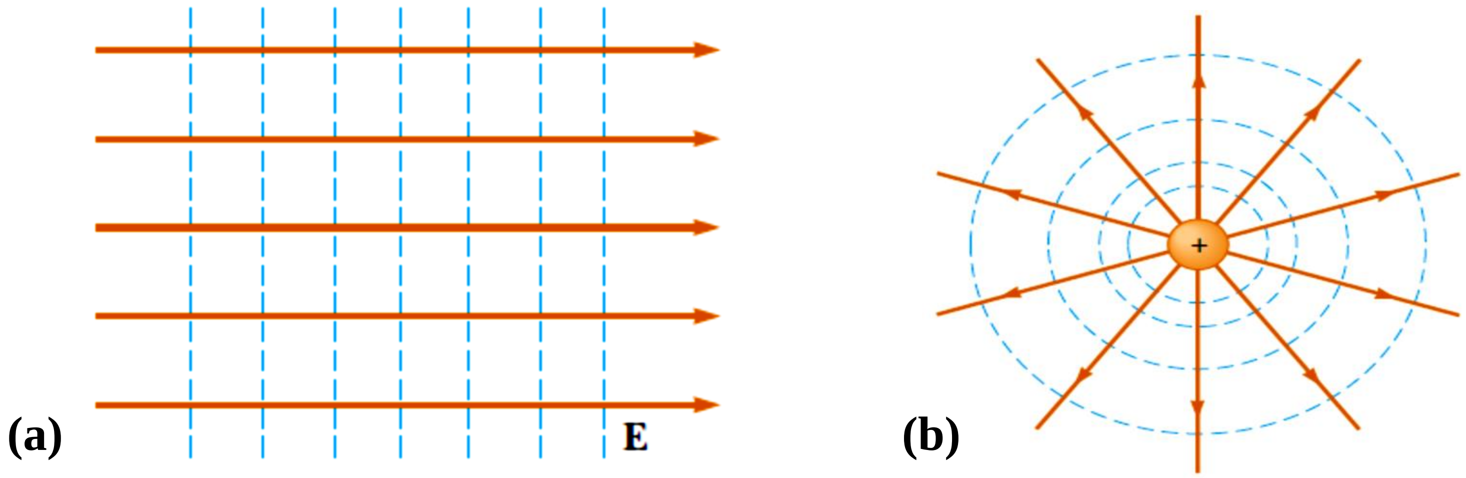


Fig. 7: Equipotential surfaces (dashed blue lines) and electric field lines (red lines) for **(a)** a uniform electric field produced by an infinite sheet of charge, **(b)** a point charge.

Electricity and Magnetism

Dr. Shurooq Saad Mahmood

Capacitance, Current and Resistance

1. Definition of Capacitance

Consider two conductors carrying charges of **equal magnitude** and **opposite sign**. Such a combination of two conductors is called a **capacitor**. The conductors are called ***plates***. The quantity of charge Q on a capacitor is **linearly proportional** to the potential difference ΔV between the conductors of the capacitor

$$Q \,\alpha\, \Delta V$$

The **proportionality constant** depends on the shape and **separation** of the conductors. We can write this relationship as

$$Q = C\,\Delta V$$

The **capacitance C** of a capacitor is defined as the ratio of the **magnitude of the charge** on either conductor to the **magnitude of the potential difference** between the conductors:

$$C = \frac{Q}{\Delta V} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The SI unit of capacitance is the **farad (F)**.

$$1\ \text{F} = 1\ \text{C/V}$$

2. Calculating the capacitance

We can calculate the capacitance for a **spherical charged conductor**, where the **electric potential** of the sphere of radius R is simply $k_e Q / R$

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

$$C = 4\pi\varepsilon_0 R$$

This expression shows that the **capacitance** of an isolated charged sphere is **proportional** to its radius and is **independent** of both the **charge on the sphere** and the **potential difference**.

The capacitance of a pair of conductors depends on the **geometry of the conductors** as following.

Parallel-Plate Capacitors

Two parallel metallic plates of equal area A are separated by a distance d, as shown in Figure 1. One plate carries a charge + Q, and the other carries a charge -Q .

The value of the electric field between two parallel plates is:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

The surface charge density on either plate is $\sigma = Q/A$

Because the **field** between the plates is **uniform**, the magnitude of the **potential difference** between the plates equals **Ed**; therefore,

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

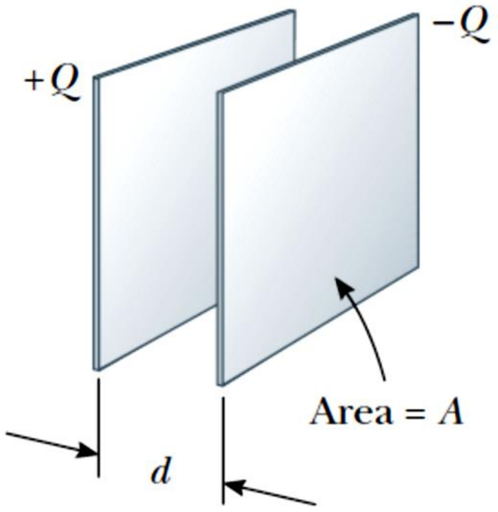


Fig.1

Substituting this result into Equation (1), we find that the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d}$$

That is, the **capacitance** of a parallel-plate capacitor is **proportional** to the area of its plates and **inversely proportional** to the plate separation.

Example 1: A parallel-plate capacitor has an area A = 2.00 x 10^{-4} m^2 and a plate separation d = 1.00 mm. Find its capacitance.

Solution:

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})\left(\frac{2.00 \times 10^{-4}\ \mathrm{m^2}}{1.00 \times 10^{-3}\ \mathrm{m}}\right)$$

$$= 1.77 \times 10^{-12}\ \mathrm{F} = 1.77\ \mathrm{pF}$$

Electricity and Magnetism

Dr. Shurooq Saad Mahmood

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$$C = 4\pi\varepsilon_0 R$$

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The surface charge density on either plate is $\sigma = Q/A$

Because the **field** between the plates is **uniform**, the magnitude of the **potential difference** between the plates equals **Ed**; therefore,

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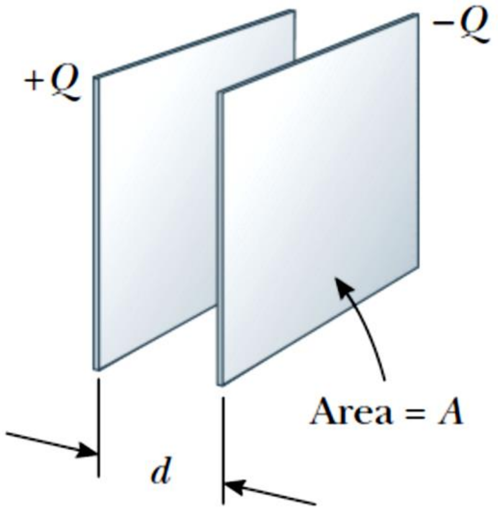

Fig.1

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$$= 1.77 \times 10^{-12} \text{ F} = 1.77 \text{ pF}$$

The capacitance of a pair of conductors depends on the **geometry of the conductors** as following:

2- The Spherical Capacitor

A **spherical capacitor** consists of a spherical conducting shell of **radius b** and charge - Q concentric with a smaller conducting sphere of **radius a** and charge + Q (Figure 2). To find the capacitance:

The **field outside a spherically symmetric charge distribution** is **radial** and given by the expression $k_e Q / r^2$.In this case, this result applies to the field between the spheres ($a < r < b$).

From Gauss's law we see that **only** the **inner sphere contributes to this field**. Thus, ***the potential difference*** between the spheres is:

$$V_b - V_a = -\int_a^b E_r\, dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r}\right]_a^b$$

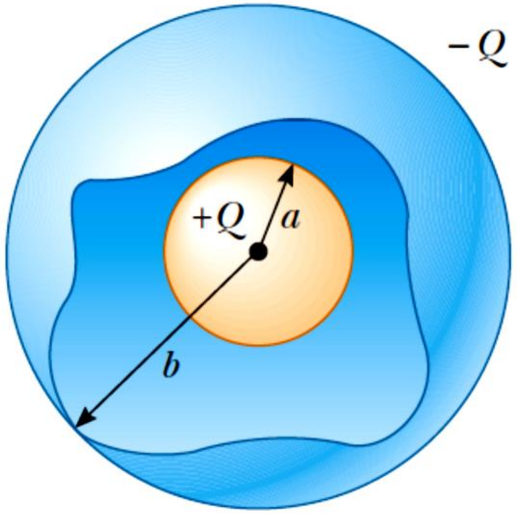


Fig. 2

2- The Spherical Capacitor

the **capacitance** approaches the value

$$= k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

The magnitude of the **potential difference** is:

$$\Delta V = |V_b - V_a| = k_e Q \frac{(b-a)}{ab}$$

Substituting this value for ΔV into Equation (1), we obtain:

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b-a)}$$

As the **radius b** of the **outer sphere** approaches infinity,

Where b >> a

$$C = \lim_{b\to\infty} \frac{ab}{K\,(b-a)} \Rightarrow \frac{ab}{K\,(b)} = \frac{a}{K}$$

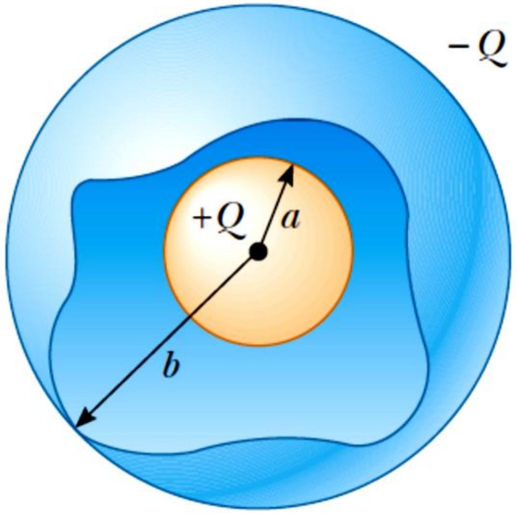


Fig. 2

2- The Spherical Capacitor

The **capacitance** approaches the value

$$C = 4\pi\varepsilon_0 a$$

3- The Cylindrical Capacitor

A **solid cylindrical conductor** of radius a and charge + Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge - Q (Figure 3).

H. W.

Find the capacitance of this cylindrical capacitor if its length is ℓ.

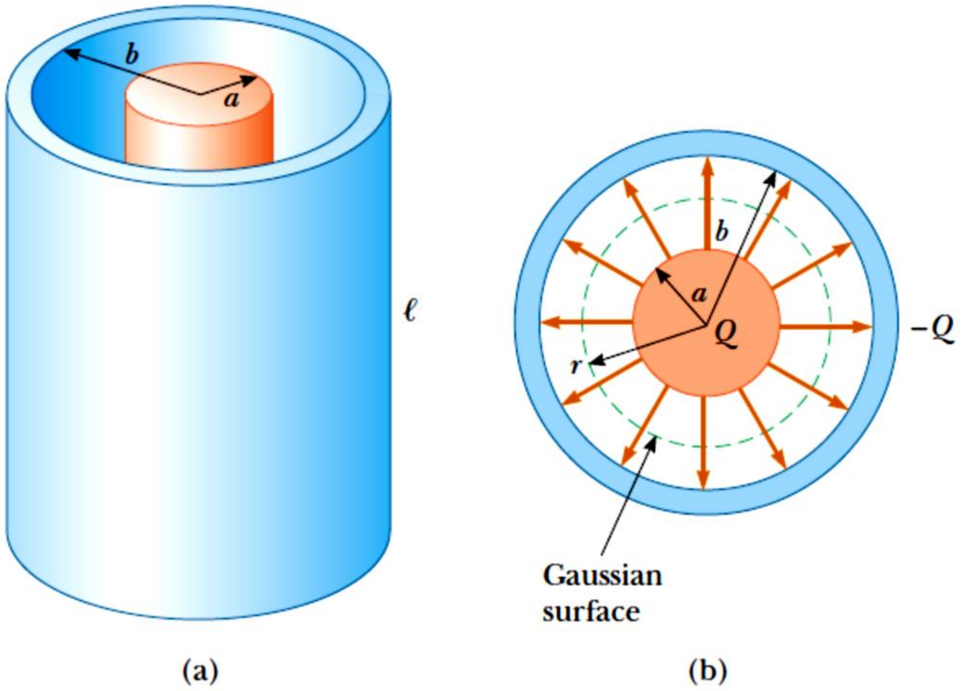


Figure 3: (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius *a* and length ℓ surrounded by a coaxial cylindrical shell of radius *b*. (b) The dashed line represents the end of the cylindrical gaussian surface of radius *r* and length ℓ.

3. Combinations of capacitors

1- Parallel Combination

Two capacitors connected as shown in Figure 4 a are known as a parallel combination of capacitors. Figure 4 b shows a circuit diagram for this combination of capacitors.

The **left plates** of the capacitors are connected by a conducting wire to the **positive terminal** of the battery and are therefore both at the **same electric potential** as the positive terminal. Likewise, the **right plates** are connected to the **negative terminal** and are therefore both at the **same potential** as the negative terminal.

Thus, the individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.

3. Combinations of capacitors

1- Parallel Combination

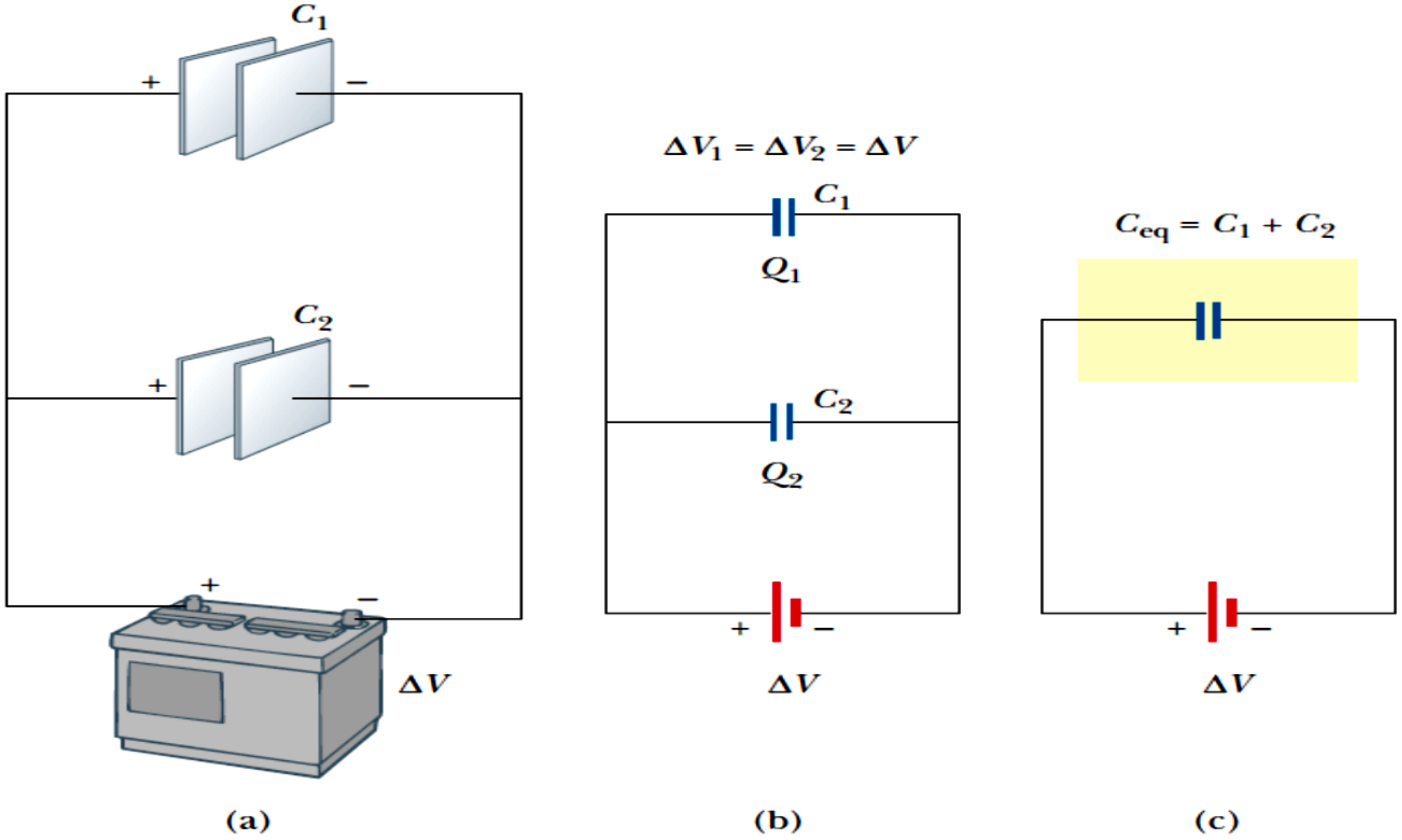


Figure 4: (a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is ΔV. (b) The circuit diagram for the parallel combination. (c) The equivalent capacitance is $C_{eq} = C_1 + C_2$.

3. Combinations of capacitors

1- Parallel Combination

The **total charge Q stored** by the two capacitors is:

$$Q = Q_1 + Q_2 \qquad \text{.................... (2)}$$

That is, the total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors. Because the voltages across the capacitors are the same, the charges that they carry are:

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

Suppose that we to replace these two capacitors by one **equivalent capacitor** having a capacitance C_{eq} , as shown in Figure 4 c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors. That is, the equivalent capacitor must store Q units of charge when connected to the battery. the voltage across the equivalent capacitor also is ΔV because the equivalent capacitor is connected directly across the battery terminals.

3. Combinations of capacitors

1- Parallel Combination

Thus, for the equivalent capacitor,

$$Q = C_{eq} \Delta V$$

Substituting these three relationships for charge into Equation 2, we have

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$

If we extend this treatment to **three or more capacitors** connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

Thus, the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitances.

3. Combinations of capacitors

2- Series Combination

Two capacitors connected as shown in Figure 5 a are known as a **series combination** of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. The other two plates are connected to each other and to nothing else.

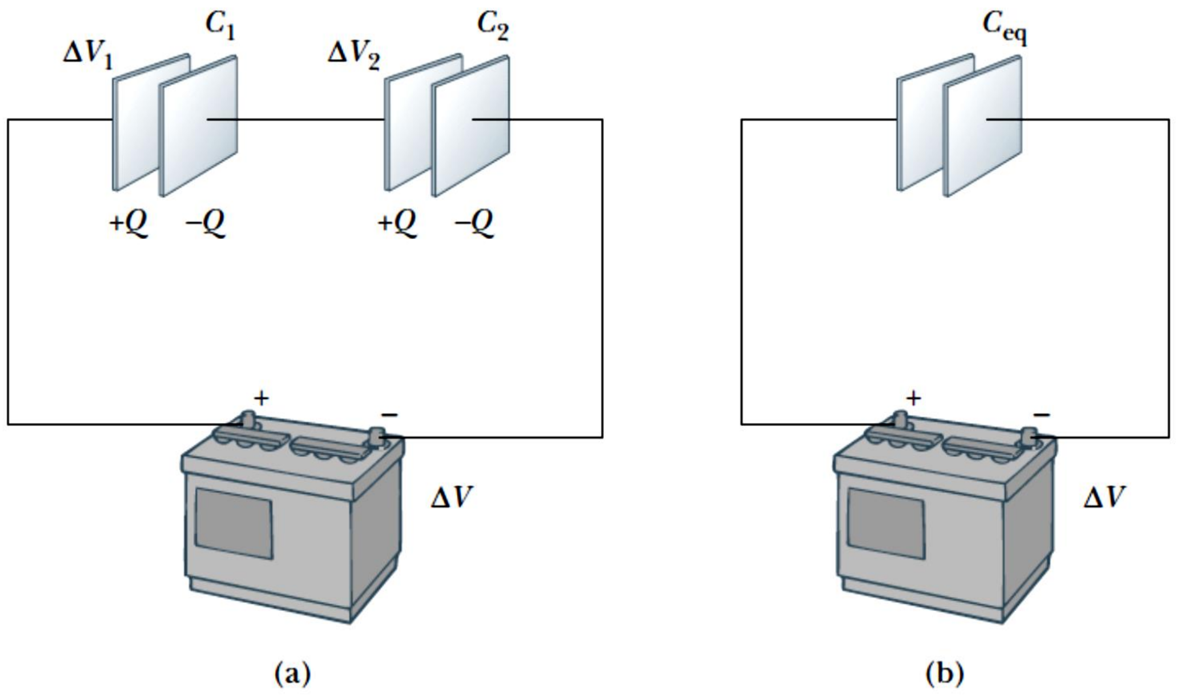


Figure 5: (a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The capacitors replaced by a single equivalent capacitor. The equivalent capacitance can be calculated from the relationship

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

3. Combinations of capacitors

2- Series Combination

Thus, the charges on capacitors connected in series are the same.

From Figure 5 a, we see that **the voltage ΔV** across the battery terminals is split between the two capacitors:

$$\Delta V = \Delta V_1 + \Delta V_2 \qquad \dots\dots\dots\dots\dots\dots\dots (3)$$

where ΔV_1 and ΔV_2 are the potential differences across capacitors C_1 and C_2 , respectively. In general, the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

Suppose that an **equivalent capacitor** has the same effect on the circuit as the series combination. After it is fully charged, the equivalent capacitor must have a charge of - Q on its right plate and a charge of + Q on its left plate. Applying the definition of capacitance to the circuit in Figure 5 b, we have

$$\Delta V = \frac{Q}{C_{eq}}$$

3. Combinations of capacitors

2- Series Combination

Because we can apply the expression $\boldsymbol{Q = C\,\Delta V}$ to each capacitor shown in Figure 5 a, the **potential difference** across each is:

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

Substituting these expressions into Equation 3 and noting that $\boldsymbol{\Delta V = Q / C_{eq}}$, we have

$$\frac{Q}{C_{\mathrm{eq}}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Canceling Q , we arrive at the relationship

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \text{(series combination)}$$

3. Combinations of capacitors

Example 2: Find the equivalent capacitance between a and b for the combination of capacitors shown in Figure 6 a. All capacitances are in microfarads.

Solution:

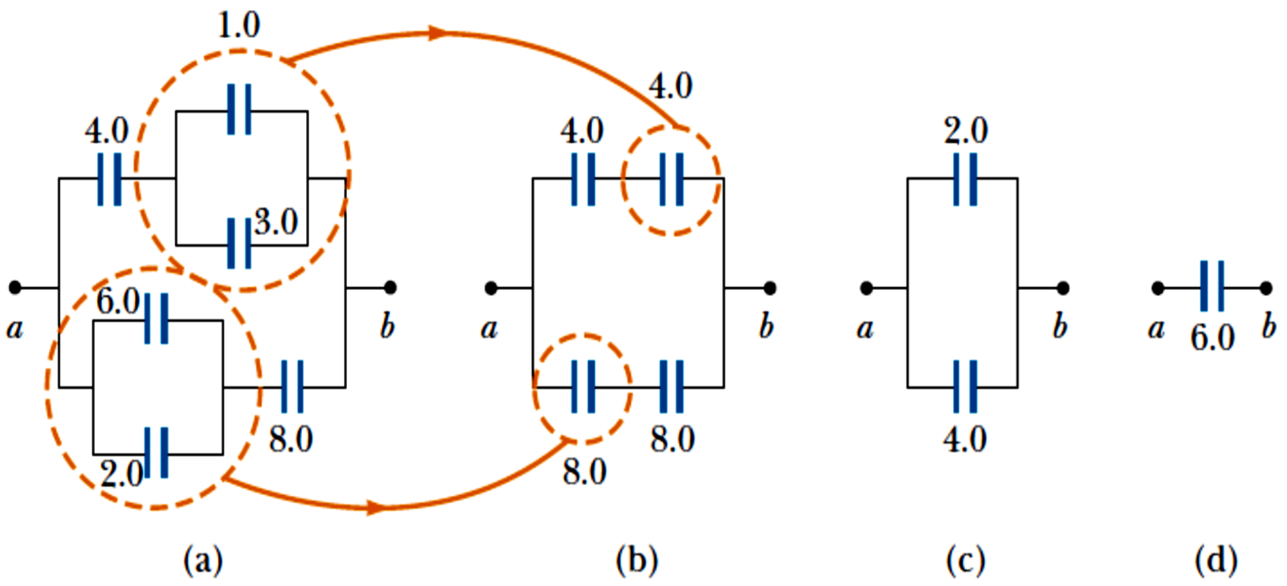


Figure 6: To find the equivalent capacitance of the capacitors in part (a), we reduce the various combinations in steps as indicated in parts (b), (c), and (d), using the series and parallel rules described in the text.

3. Combinations of capacitors

we reduce the combination step by step as indicated in the figure. The 1.0 μF and 3.0 μF capacitors are in parallel and combine according to the expression:

$$C_{eq1} = C_1 + C_2 \quad = 1\ \mu F + 3\ \mu F = 4\mu F$$

The 2.0 μF and 6.0 μF capacitors also are in parallel and have an equivalent capacitance of:

$$C_{eq2} = C_4 + C_5 \quad = 6\ \mu F + 2\ \mu F = 8\mu F$$

Thus, the upper branch in Figure 6 b consists of two 4.0 μF capacitors in series, which combine as follows:

$$1/C_{eq3} = 1/C_3 + 1/C_{eq1} \quad = 1/4 + 1/4 \quad C_{eq3} = 2\ \mu F$$

The lower branch in Figure 6 b consists of two 8.0 μF capacitors in series, which combine to yield an equivalent capacitance of :

$$1/C_{eq4} = 1/C_{eq2} + 1/C_6 \quad = 1/8 + 1/8 \quad C_{eq4} = 4\ \mu F$$

Finally, the 2.0 μF and 4.0 μF capacitors in Figure 6 c are in parallel and thus have an equivalent capacitance of

$$C_{total} = C_{eq3} + C_{eq4} \quad = 2\mu F + 4\mu F = 6\ \mu F$$

4. Electric current

Consider a system of **electric charges in motion**. Whenever there is a **net flow of charge** through some region, a current is said to exist. To define current more precisely, suppose that the **charges are moving** perpendicular **to a surface of area A**, as shown in Figure 7.

The current is the rate at which charge flows through this surface.

If **ΔQ** is the amount of **charge** that passes through this area in a **time** interval **Δt**, the average current I_{av} is equal to the charge that passes through A per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

If the rate at which **charge flows varies in time**, then the **current varies in time**; we define the **instantaneous current I** as the differential limit of average current:

$$I \equiv \frac{dQ}{dt}$$

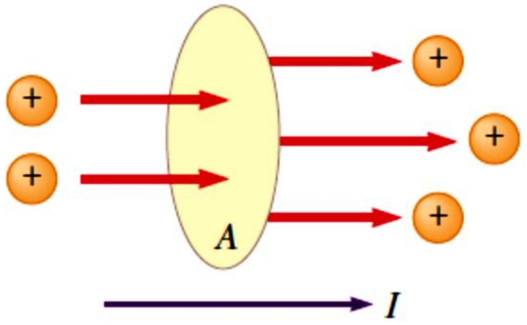


Figure 7: Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

4. Electric current

The SI unit of current is the **ampere (A)**:

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

The charges passing through the surface in Figure 7 can be positive or negative, or both. **It is conventional to assign to the current the same direction as the flow of positive charge**.

The direction of the current is opposite the direction of flow of electrons, while **the current the same direction as the flow of positive charge.**

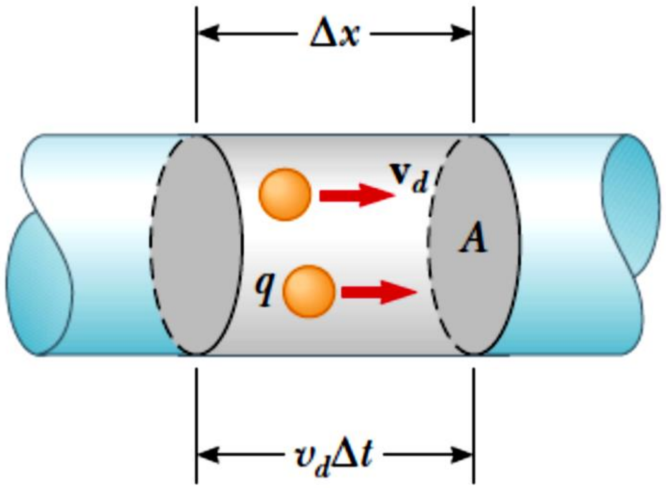
4. Electric current

Consider the current in a conductor of cross-sectional area A (Figure 8). The **volume** of a section of the conductor of **length** Δx (the gray region shown in Fig. 8) is $\boldsymbol{A\,\Delta x}$. If n represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is $\boldsymbol{nA\,\Delta x}$. Therefore, the charge ΔQ in this section is

$$\Delta Q = \textbf{number of carriers in section x charge per carrier} = (nA\,\Delta x)q$$

where q is the charge on each carrier.

Figure 8: A section of a uniform conductor of cross-sectional area *A*. The mobile charge carriers move with a speed v_d , and the distance they travel in a time Δt is $\Delta x = v_d\,\Delta t$. The number of carriers in the section of length Δx is $nAv_d\,\Delta t$, where n is the number of carriers per unit volume.



4. Electric current

If the carriers move with a **speed v_d**, the distance they move in a time Δt is $\Delta x = v_d \Delta t$. Therefore, we can write ΔQ in the form

$$\Delta Q = (nAv_d \Delta t)q$$

If we divide both sides of this equation by Δt , we see that the average current in the conductor is

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_d A$$

The **current density J** in the conductor is defined as **the current per unit area**

$$J = \frac{I}{A} = nqv_d \qquad \text{A/m}^2$$

Where v_d is the drift speed of the charge carriers, we see that **current density** is in the **direction of charge motion for positive charge carriers** and opposite the **direction of motion for negative charge carriers**.

Electricity and Magnetism

Dr. Shurooq Saad Mahmood

Capacitance, Current and Resistance

5. Resistance and Ohm's Law

In Chapter (Gauss's Law) we found that **no electric field can exist inside a conductor**. However, this statement is **true only** if the conductor is in static equilibrium. This section is describe what happens when the charges in the conductor are **allowed to move**.

Charges moving in a conductor produce a **current** under the action of an electric field. An **electric field can exist** in the **conductor** because the charges in this situation are in motion, that is, this is a *nonelectrostatic* situation.

Consider a conductor of cross-sectional **area A** carrying a **current I**. The **current density J** in the conductor is defined as the current per unit area. Because the current $\boldsymbol{I = nqv_dA}$, the current density is

$$J \equiv \frac{I}{A} = nqv_d$$

where J has SI units of **A/m²**. This expression is **valid only** if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current.

5. Resistance and Ohm's Law

In general, the **current density** is a vector quantity:

$$\mathbf{J} = nq\mathbf{v}_d$$

From this equation, we see that **current density**, **like current**, is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor.

If the **potential difference** is constant, then the **current** also is constant. In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma\mathbf{E} \qquad \text{.....................} \ (1)$$

where the constant of proportionality σ is called the **conductivity** of the conductor. Materials that obey Equation (1) are said to follow **Ohm's law**, named after Georg Simon Ohm (1787–1854).

5. Resistance and Ohm's Law

More specifically, ***Ohm's law*** states that: for many materials (including most **metals**), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

We can obtain a form of Ohm's law useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length ℓ, as shown in Figure 9. A potential difference $\boldsymbol{\Delta V = V_b - V_a}$ is **maintained across the wire**, creating in the wire an **electric field** and a **current**. If the **field** is assumed to be uniform, the potential difference is related to the field through the relationship

$$\Delta V = E\ell$$

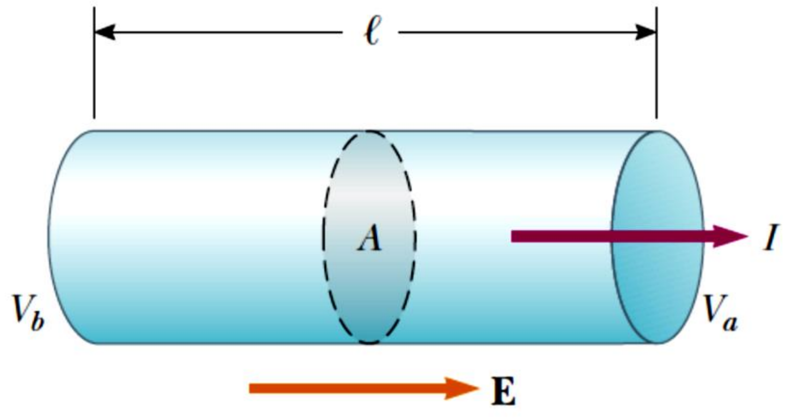


Figure 9: A uniform conductor of length ℓ and cross-sectional area A. A potential difference $\boldsymbol{\Delta V = V_b - V_a}$ maintained across the conductor sets up an electric field **E**, and this field produces a current I that is proportional to the potential difference.

5. Resistance and Ohm's Law

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

Because ***J* = *I/A***, we can write the **potential difference** as

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I$$

The quantity $\ell / \sigma A$ is called the resistance **R** of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current through the conductor:

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I} \qquad \text{…………….. (2)}$$

From this result we see that resistance has SI units of **volts per ampere**. One volt per ampere is defined to be **1 ohm (Ω)**:

$$1\,\Omega \equiv \frac{1\text{ V}}{1\text{ A}}$$

5. Resistance and Ohm's Law

The inverse of conductivity is resistivity ρ :

$$\rho \equiv \frac{1}{\sigma}$$

where ρ has the units **ohm-meters** (Ω . m). We can use this definition and Equation (2) to express the **resistance of a uniform block of material** as

$$R = \rho \frac{\ell}{A} \qquad \text{………..} \ (3)$$

6. Electrical Energy And Power

The rate at which the charge Q loses potential energy in going through the resistor is

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} \Delta V = I \Delta V$$

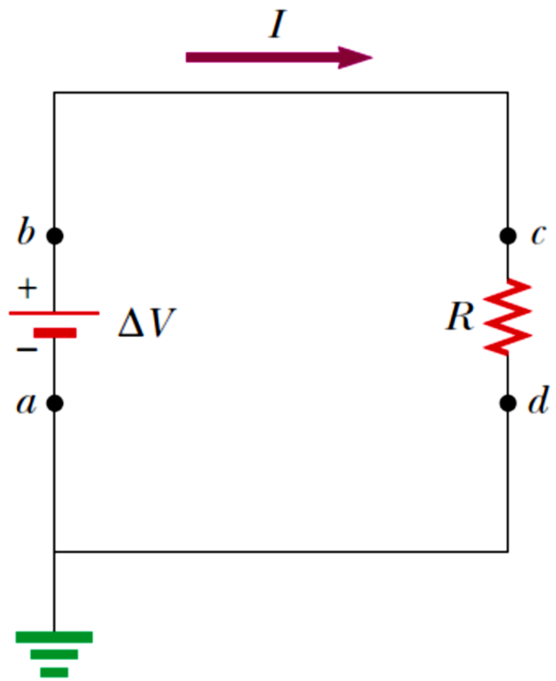


Figure 10: A circuit consisting of a resistor of resistance R and a battery having a potential difference ΔV across its terminals. Positive charge flows in the clockwise direction.

6. Electrical Energy And Power

where $\Delta U = q\,\Delta V$ and I is the current in the circuit.

Thus, the **power P**, representing the **rate at which energy is delivered to the resistor**, is

$$P = I\,\Delta V \qquad \text{.............} \ (3)$$

Using Equation 3 and the fact that $\Delta V = IR$ for a resistor, we can express the power delivered to the resistor in the alternative forms:

$$P = I^2 R = \frac{(\Delta V)^2}{R}$$

TABLE (1) Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] $(\Omega \cdot m)$	Temperature Coefficient $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[b]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\approx 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C.

[b] A nickel–chromium alloy commonly used in heating elements.

Example 3: Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of 2.00×10^{-4} m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of 3×10^{10} Ω . m.

Solution:

From Equation 3 and Table 1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= 1.41 \times 10^{-5}\ \Omega$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= 1.5 \times 10^{13}\ \Omega$$

Example 4: (a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution:

The cross-sectional area of this wire is:

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega\ .\ \text{m}$ (see Table 1). Thus, we can use Equation 1 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega \cdot \text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

(b) If a potential difference of 10 V is maintained across a 1.0 m length of the Nichrome wire, what is the current in the wire?

Solution:

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

Example 5: An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Solution:

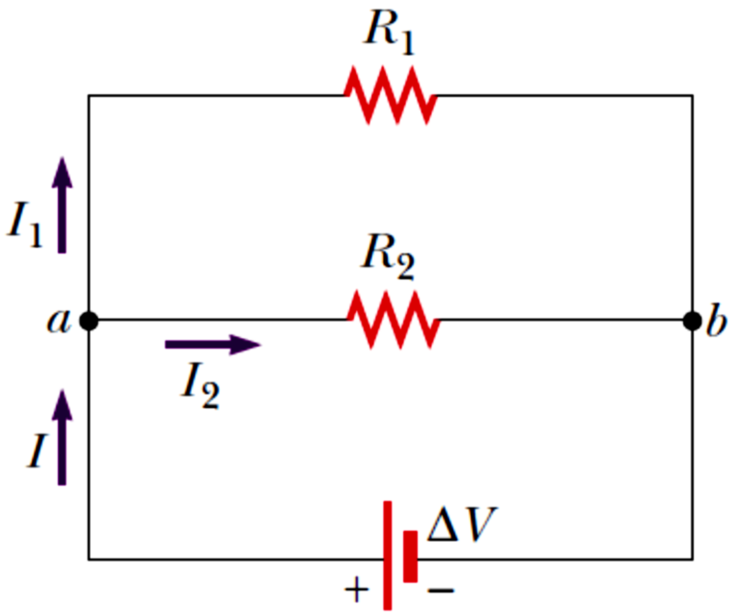
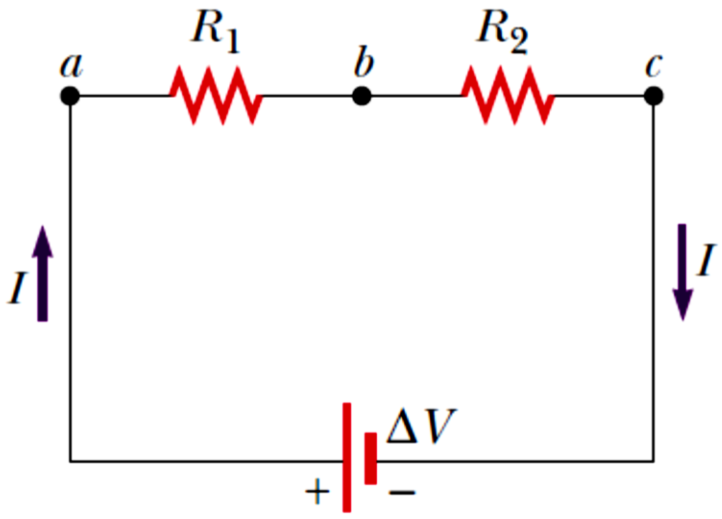
Because ΔV = IR, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

We can find the power rating using the expression

$$\mathcal{P} = I^2R = (15.0\ \text{A})^2(8.00\ \Omega) = \boxed{1.80\ \text{kW}}$$

7. Resistors in Series and Parallel

In parallel combination	In series combination
	
The potential differences across the resistors are the same. $V_{total}=V_1=V_2$	The potential difference applied across the series combination of resistors will divide between the resistors. $V_{total}=V_1+V_2$

7. Resistors in Series and Parallel

In parallel combination	In series combination
The current I that enters point a must equal the total current leaving that point. $I_{total} = I_1 + I_2$	The current results in the same current in the battery as $I_{total} = I_1 = I_2$
The equivalent resistance is $1/R_{eq} = 1/R_1 + 1/R_2$	The equivalent resistance is $R_{eq} = R_1 + R_2$

Example 6: Three resistors are connected in parallel as shown in Figure 11. A potential difference of 18 V is maintained between points a and b. **(a)** Find the current in each resistor.

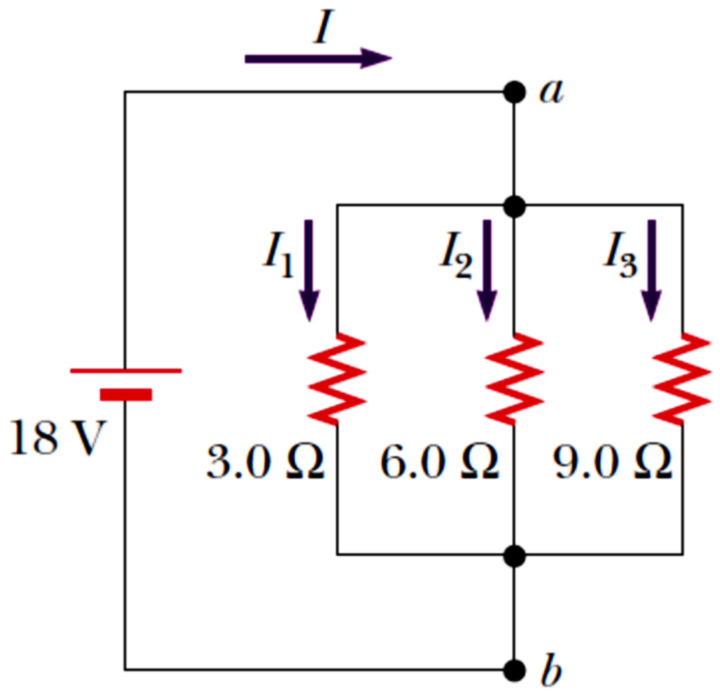
Solution:

The resistors are in parallel, and so the potential difference across each must be 18 V. Applying the relationship ΔV = IR to each resistor gives

$$I_1 = \frac{\Delta V}{R_1} = \frac{18\ \text{V}}{3.0\ \Omega} = \boxed{6.0\ \text{A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\ \text{V}}{6.0\ \Omega} = \boxed{3.0\ \text{A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\ \text{V}}{9.0\ \Omega} = \boxed{2.0\ \text{A}}$$



Example 6: (b) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

Solution:

We apply the relationship $\mathcal{P} = (\Delta V)^2/R$ to each resistor and obtain

$$\mathcal{P}_1 = \frac{\Delta V^2}{R_1} = \frac{(18\text{ V})^2}{3.0\ \Omega} = 110\text{ W}$$

$$\mathcal{P}_2 = \frac{\Delta V^2}{R_2} = \frac{(18\text{ V})^2}{6.0\ \Omega} = 54\text{ W}$$

$$\mathcal{P}_3 = \frac{\Delta V^2}{R_3} = \frac{(18\text{ V})^2}{9.0\ \Omega} = 36\text{ W}$$

This shows that the smallest resistor receives the most power. Summing the three quantities gives a total power of 200 W.

Example 6: (c) Calculate the equivalent resistance of the circuit.

Solution:

$$\frac{1}{R_{eq}} = \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} + \frac{1}{9.0\ \Omega}$$

$$= \frac{6}{18\ \Omega} + \frac{3}{18\ \Omega} + \frac{2}{18\ \Omega} = \frac{11}{18\ \Omega}$$

$$R_{eq} = \frac{18\ \Omega}{11} = \boxed{1.6\ \Omega}$$

Electricity and Magnetism

Dr. Shurooq Saad Mahmood

Magnetostatics: Magnetic Field and Magnetic Flux

Magnetostatics and Magnetic Field

Magnetostatics is the subfield of electromagnetics describing a **static magnetic fields**, it study of **magnetic fields** in systems where the **currents are steady** (not changing with time). It is the magnetic analogue of electrostatics, where **the charges are stationary**.

Every magnet, regardless of its shape, has **two poles**, called north (N) and south (S) poles as shown in Figure 1. The **exert forces on other magnetic poles** similar to the way that **electric charges exert forces on one another**, where the different magnetic electrodes are attracted to each other (N - S), while the **similar** polynomials (N - N or S - S) are **repulsion**.

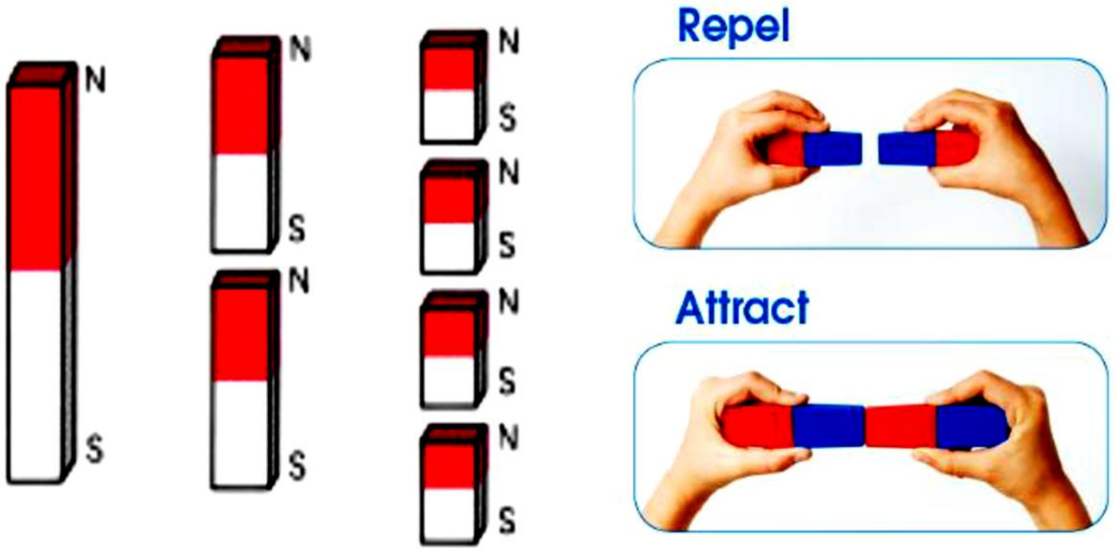


Fig 1

Magnetostatics and Magnetic Field

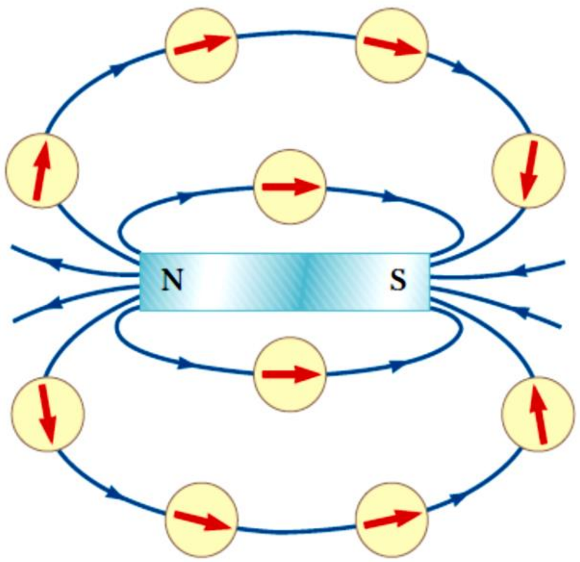
The magnetic field (B) is a vector that has both **magnitude and direction**. The direction of the magnetic field at any point in space is the direction indicated by the **north pole** of a small compass needle placed at that point (Figure 2).

The magnetic field **B** on the test object at some point in space in terms can be **determined** by a magnetic force $\mathbf{F_B}$ that the field exerts on a charged particle moving with a velocity **v**. Let us assume that no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

(Properties of the magnetic force on a charge moving in a magnetic field B)

- The **magnitude** F_B of the **magnetic force** exerted on the particle is proportional to the **charge** q and to the **speed** v of the particle.
- **The magnitude and direction of** F_B depend on the **velocity** of the particle and on the **magnitude and direction of the magnetic field B**.

Fig 2

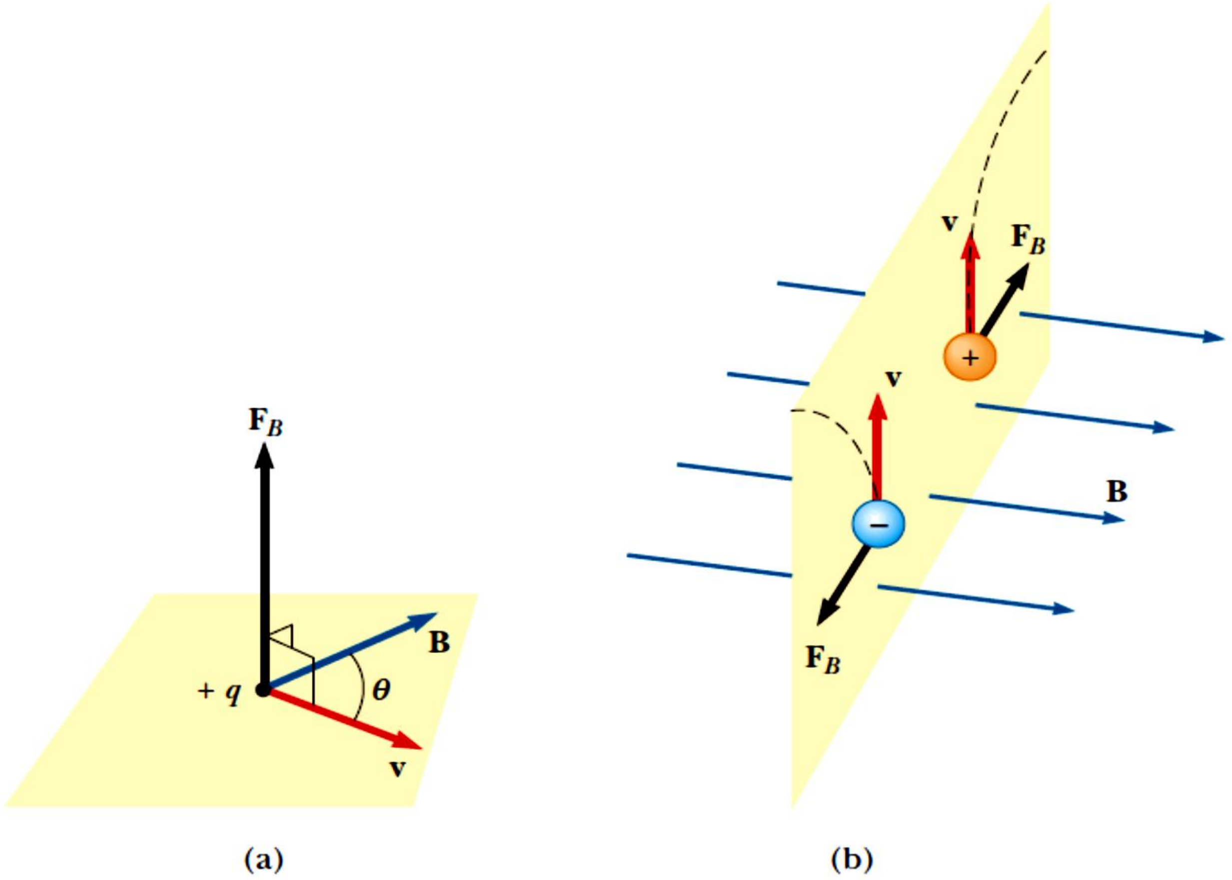


Magnetostatics and Magnetic Field

• When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is **zero**.

• When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and **B**; that is, $\mathbf{F_B}$ is perpendicular to the plane formed by **v** and **B** (Figure 3a).

Figure 3: The direction of the magnetic force $\mathbf{F_B}$ acting on a charged particle moving with a velocity **v** in the presence of a magnetic field **B**. (a) The magnetic force is perpendicular to both **v** and **B**. (b) Oppositely directed magnetic forces $\mathbf{F_B}$ are exerted on two oppositely charged particles moving at the same velocity in a magnetic field.



Magnetostatics and Magnetic Field

• The magnetic force exerted on a **positive charge** is in the direction **opposite** the direction of the magnetic force exerted on a **negative charge** moving in the same direction (Figure 3b).

• The magnitude of the magnetic force exerted on the moving particle is proportional to sin θ, where θ is the angle the particle's **velocity vector** makes with the direction of **B**.

We can summarize these observations by writing the **magnetic force** in the form

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

The magnitude of the magnetic force on a charged particle is

$$F_B = |q| vB \sin\theta$$

where θ is the angle between **v** and **B**. From this expression, we see that **F_B is zero** when **v** is **parallel** or **antiparallel** to **B** (θ = 0 or 180°) and maximum when **v** is **perpendicular** to **B** (θ = 90°).

Magnetostatics and Magnetic Field

To determine the direction of the force on a positive charge, we use a **righthand rule** that helps us understand the 3D perpendicular nature of magnetic fields.

Figure 4 reviews the **right-hand rule** for determining the direction of the cross product $\mathbf{v} \times \mathbf{B}$. You point the four fingers of your right hand along the direction of **v** with the palm facing **B** and curl them toward **B**. The extended thumb, which is at a right angle to the fingers, points in the direction of $\mathbf{v} \times \mathbf{B}$. Because $\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$ is in the direction of $\mathbf{v} \times \mathbf{B}$ if q is **positive** (Fig. 4a) and opposite the direction of $\mathbf{v} \times \mathbf{B}$ if q is **negative** (Fig. 4b).

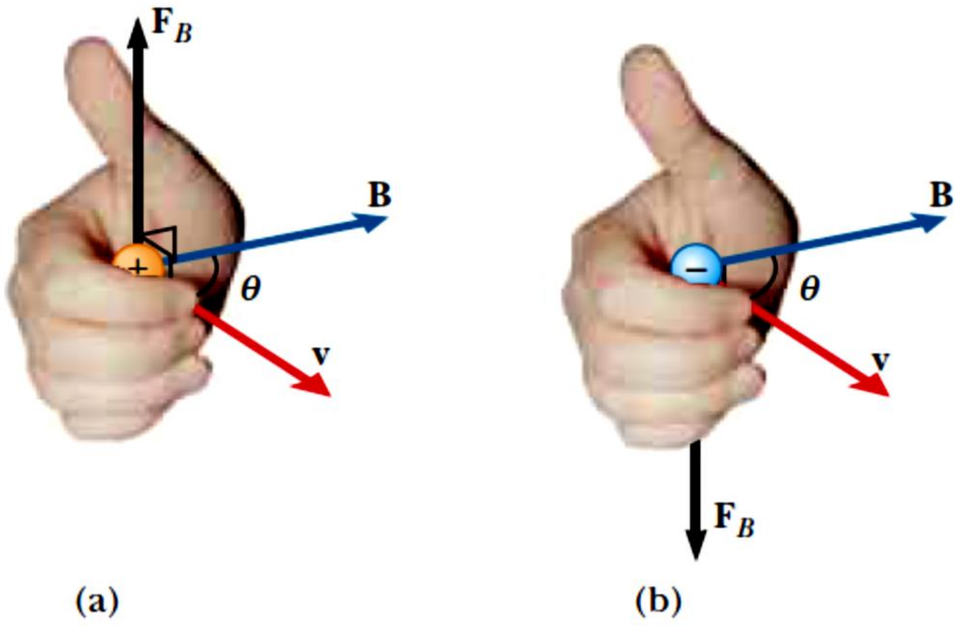


Figure 4: The right-hand rule for determining the direction of the magnetic force $\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$ acting on a particle with charge q moving with a velocity **v** in a magnetic field **B**.
The direction of $\mathbf{v} \times \mathbf{B}$ is the direction in which the thumb points. (a) If q is **positive**, $\mathbf{F}_B$ is upward. (b) If q is **negative**, $\mathbf{F}_B$ is downward, antiparallel to the direction in which the thumb points.

Magnetostatics and Magnetic Field

Note: When a charge is placed in a magnetic field, it experiences a magnetic force if two conditions are met:

1. The charge must be **moving**. No magnetic force acts on a stationary charge.

2. The **velocity of the moving charge** must have a component that is **perpendicular** to the direction of the **field**.

Unit of Magnetic Field

The SI unit of magnetic field is the **newton per coulomb-meter per second**, which is called the **tesla (T)**:

$$1\ \mathrm{T} = \frac{\mathrm{N}}{\mathrm{C \cdot m/s}}$$

Because a **coulomb per second** is defined to be an **ampere**, we see that

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A \cdot m}}$$

A non-SI magnetic-field unit in common use, called the **gauss (G)**, is related to the tesla through the conversion $1\mathrm{T}=10^4\ \mathrm{G}$.

The properties of magnetic field line

1. The lines **originate** from the **North** Pole and end on the **South** Pole.

2. The **magnetic field** at any point is **tangent** to the **magnetic field line** at that point.

3. The **strength** of the field is proportional to the **number of lines per unit area** that passes through a surface oriented **perpendicular** to the **lines**.

4. The **magnetic field lines** will never come to cross each other.

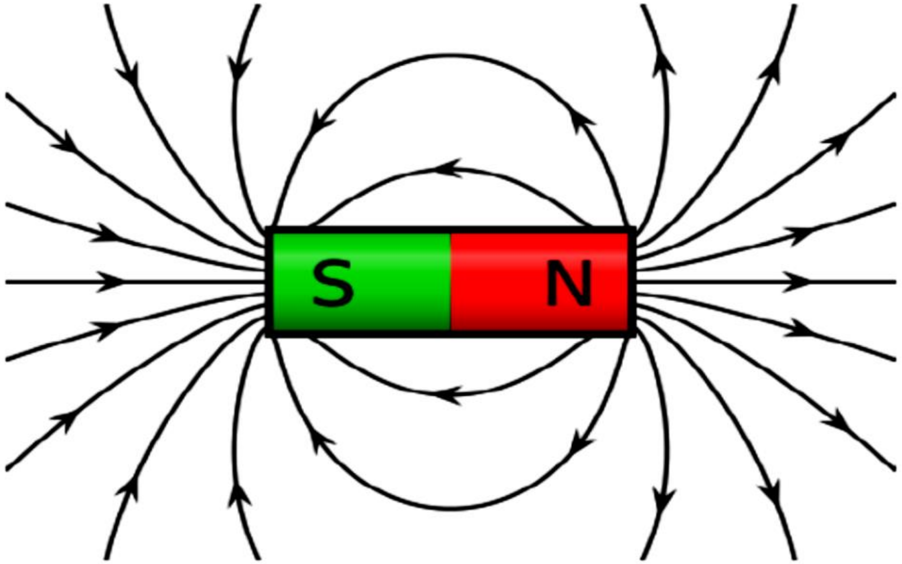


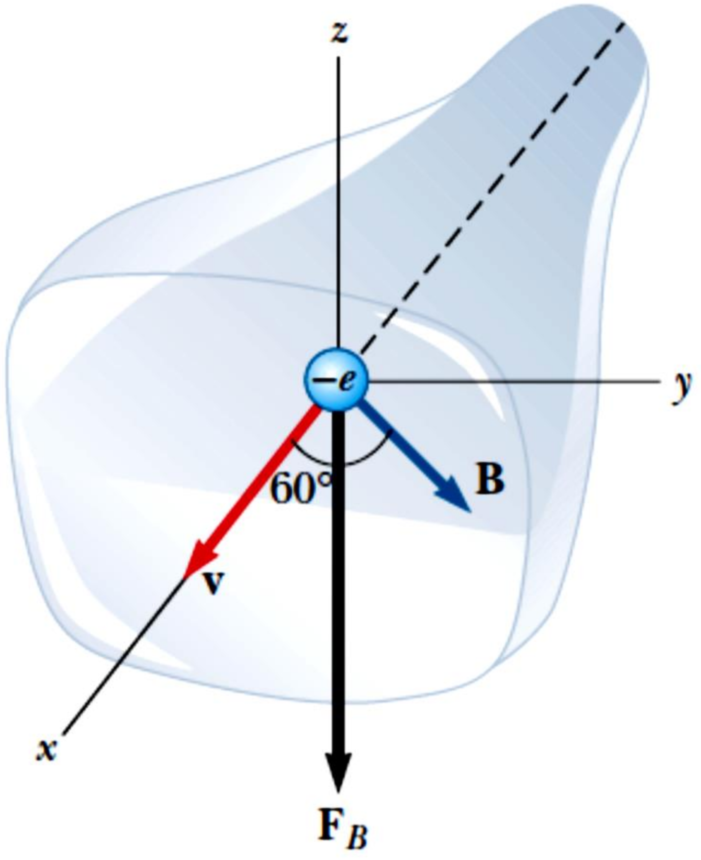
Figure 5: The magnetic field lines of a bar magnetic

Example 1: An electron in a television picture tube moves toward the front of the tube with a speed of $\mathbf{8 \times 10^6}$ **m/s** along the x axis (Fig. 29.5). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude **0.025 T**, directed at an angle of **60°** to the x axis and lying in the **xy** plane. Calculate the magnetic force on and acceleration of the electron.

Solution:

$$F_B = |q|vB\sin\theta$$
$$= (1.6 \times 10^{-19}\ \text{C})(8.0 \times 10^6\ \text{m/s})(0.025\ \text{T})(\sin 60°)$$
$$= \boxed{2.8 \times 10^{-14}\ \text{N}}$$

Because **v** × **B** is in the **positive** z direction (from the righthand rule) and the charge is **negative**, $\mathbf{F_B}$ is in the **negative** z direction.



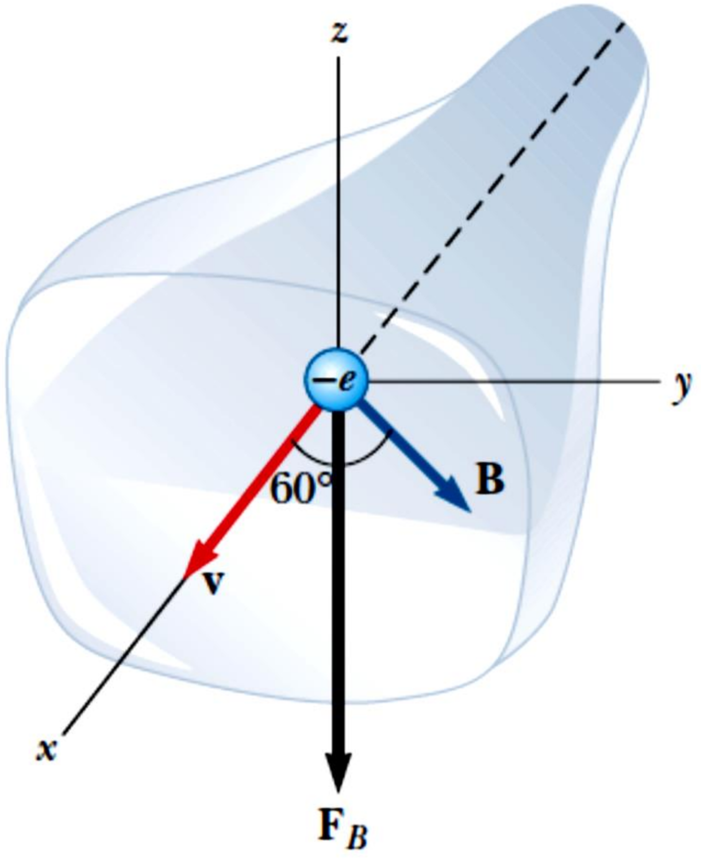
Example 1: An electron in a television picture tube moves toward the front of the tube with a speed of $\mathbf{8 \times 10^6}$ **m/s** along the x axis (Fig. 29.5). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude **0.025 T**, directed at an angle of **60°** to the x axis and lying in the **xy** plane. Calculate the magnetic force on and acceleration of the electron.

Solution:

The mass of the electron is 9.11×10^{-31} kg, and so its **acceleration** is

$$a = \frac{F_B}{m_e} = \frac{2.8 \times 10^{-14}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}} = 3.1 \times 10^{16}\ \text{m/s}^2$$

in the **negative** z direction.



Electricity and Magnetism

Dr. Shurooq Saad Mahmood

Magnetostatics: Ampère's Law and Magnetic Flux

The Biot – Savart Law

Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the **force exerted** by an **electric current** on a **nearby magnet**. From their experimental results, Biot and Savart arrived at a mathematical expression that gives **the magnetic field at some point in space in terms of the current that produces the field**. That expression is based on the following experimental observations for the magnetic field $d\mathbf{B}$ at a point P associated with a length element $d\mathbf{s}$ of a wire carrying a steady current I (Figure 1):

((Properties of the magnetic field created by an electric current))

- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ to P.

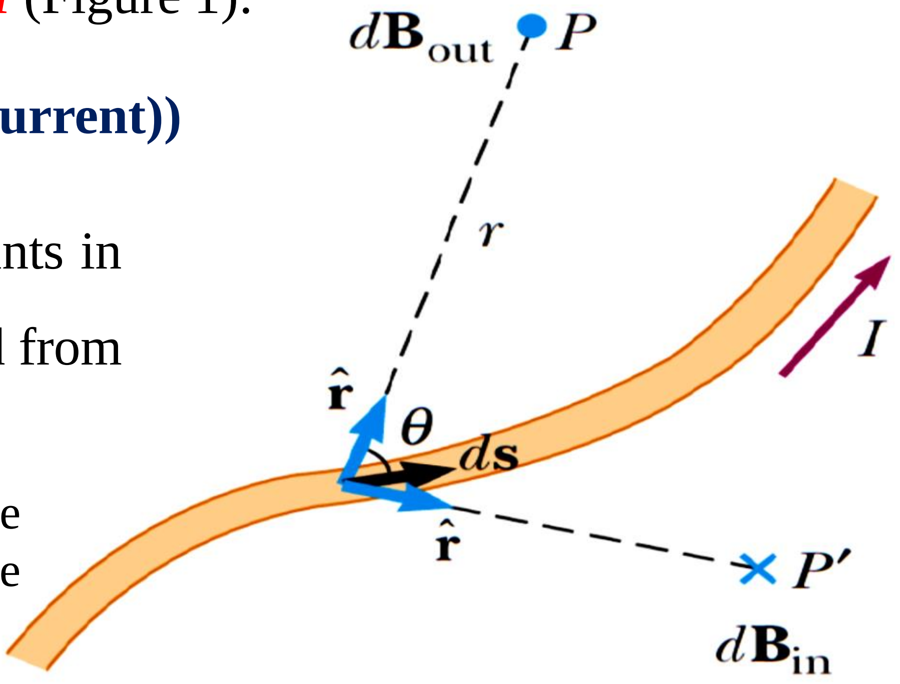


Figure 1: The magnetic field $d\mathbf{B}$ at point P due to the current I through a length element $d\mathbf{s}$ is given by the **Biot–Savart law**.

The Biot – Savart Law

- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is proportional to the **current** and to the **magnitude** ds of the length element $d\mathbf{s}$.
- The magnitude of $d\mathbf{B}$ is proportional to **sin θ** where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{\mathbf{r}}$.

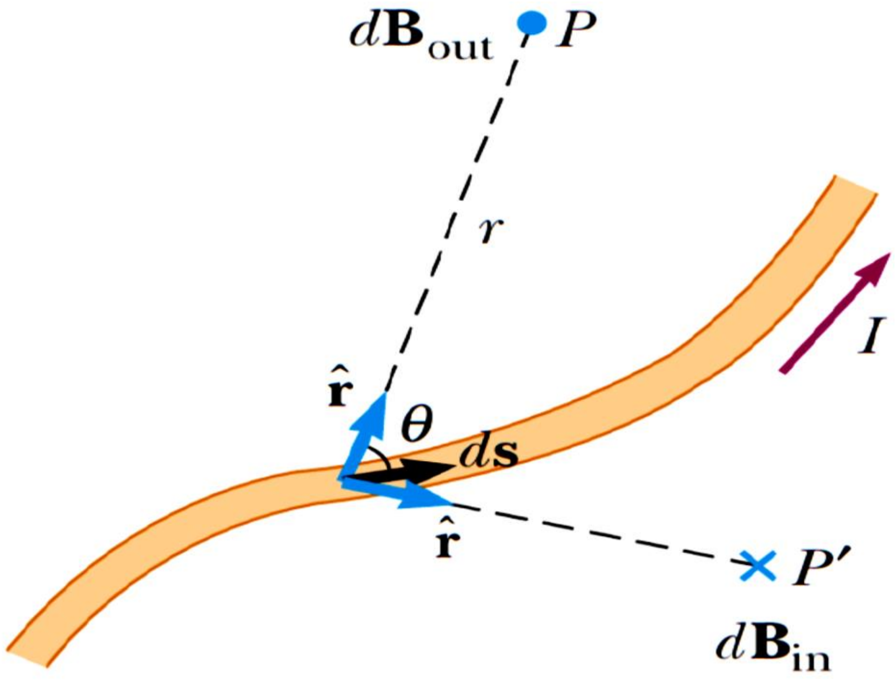


Figure 1: The magnetic field $d\mathbf{B}$ at point P due to the current I through a length element $d\mathbf{s}$ is given by the **Biot–Savart law**.

The Biot – Savart Law

These observations are summarized in the **mathematical formula** known today as the **Biot–Savart law**:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \qquad \text{………………… (1)}$$

where μ_0 is a constant called the **permeability of free space**, and has a value of:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

It is important to note that the field $d\mathbf{B}$ in Equation (1) is the field created by the current in only a small length element $d\mathbf{s}$ of the conductor. To find the total magnetic field **B** created at some point by a current of finite size, we must sum up contributions from all current elements $Id\mathbf{s}$ that make up the current. The **total magnetic field** is getting by integral Equation (1):

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

Ampère's Law

The **magnetic field** at a distance ***r*** from a **very long straight wire** (Figure 2), carrying a steady current ***I*** (i.e. do not change with time), has a magnitude equal to:

$$B = \frac{\mu_0 I}{2\pi r}$$

and a direction perpendicular to ***r*** and ***I***. The closed path integral along a circle centered around the wire, which is equivalent to the line integral of **B** . *d***s**, is

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

Where $\oint ds = 2\pi r$ is the **circumference** of the circular path.

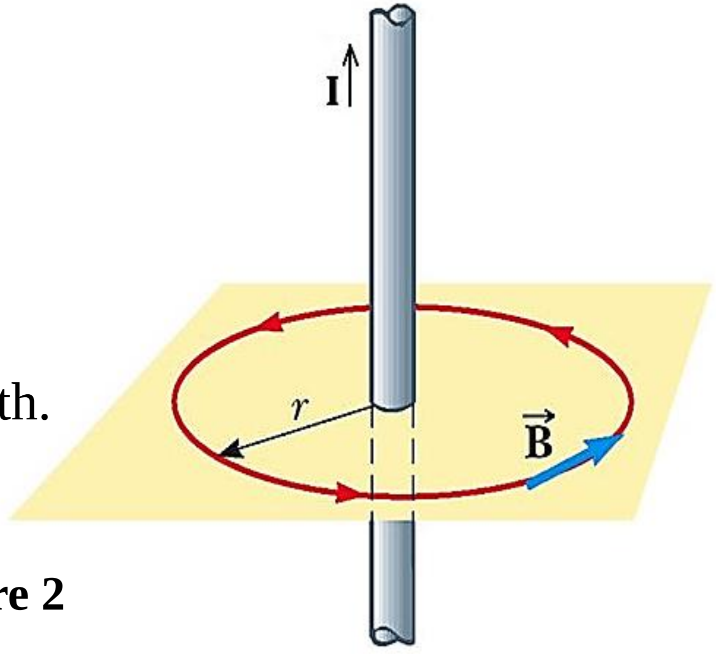


Figure 2

Ampère's Law

The general case, known as **Ampère's law**, can be stated as follows:

The line integral of **B** . *d***s** around any closed path equals $\mu_0 I$, where *I* is the total continuous current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

Note: In order to apply Ampère's Law all **currents** have to be steady (i.e. **do not change with time**).

Magnetic Flux

The **flux** associated with a **magnetic field** is defined in a manner similar to that used to define **electric flux**.

$$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$$

Consider an element of area $d\mathbf{A}$ on an **arbitrarily shaped surface**, as shown in (Figure 3). If the **magnetic field** at this element is **B**, the magnetic flux through the element is $\mathbf{B} \cdot d\mathbf{A}$, where $d\mathbf{A}$ is a vector that is perpendicular to the **surface** and has a magnitude equal to the area dA.

Hence, the **total magnetic flux** Φ_B through the surface $d\mathbf{A}$ is:

$$\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A}$$

The unit of **flux** is the $\mathbf{T \cdot m^2}$, which is defined as a **weber** (Wb); 1 Wb = 1 $\text{T} \cdot \text{m}^2$.

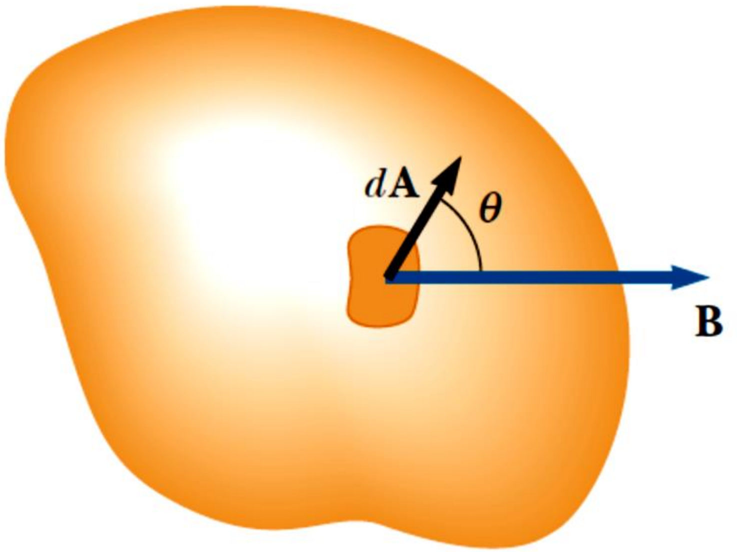


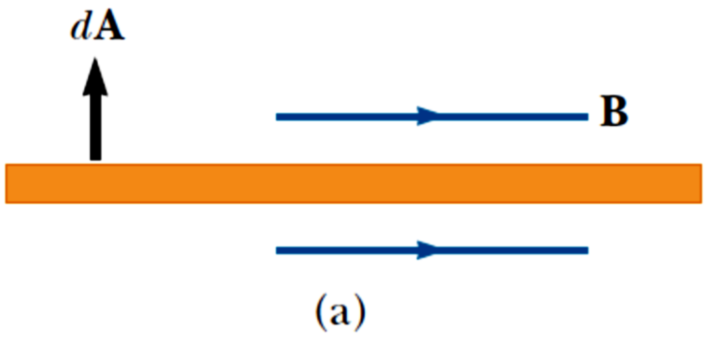
Figure 3: The **magnetic flux** through an **area** element $d\mathbf{A}$ is $\mathbf{B} \cdot d\mathbf{A} = BdA \cos\theta$, where $d\mathbf{A}$ is a vector perpendicular to the surface.

Magnetic Flux

Consider the special case of a plane of area **A** in a uniform field **B** that makes an **angle θ** with *d***A**. The **magnetic flux** through the plane in this case is:

$$\Phi_B = BA\cos\theta$$

- If the **magnetic field** is parallel to the plane, as in Figure 4a, then **θ=90°** and the **flux** is zero.
- If the **magnetic field** is perpendicular to the plane, as in Figure 4b, then **θ=0** and the **flux** is ***BA*** (the **maximum value**).



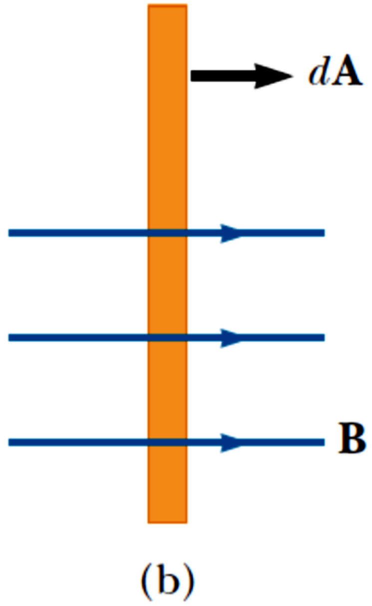


Figure 4: Magnetic flux through a plane lying in a magnetic field.
(a) The **flux** through the plane is zero when the **magnetic field** is **parallel** to the plane surface.
(b) The **flux** through the plane is a maximum when the **magnetic field** is perpendicular to the plane.

3. Electric Potential and Potential Energy Due to Point Charges

Example 3: A charge $\boldsymbol{q_1}$ **= 2.00 μC** is located at the origin, and a charge $\boldsymbol{q_2}$ **= - 6.00 μC** is located at (0, 3.00) m, as shown in Figure 6a. **(a)** Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

Solution:

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 8.99 \times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= -6.29 \times 10^3\ \mathrm{V}$$

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = 5\ \mathrm{m}$$

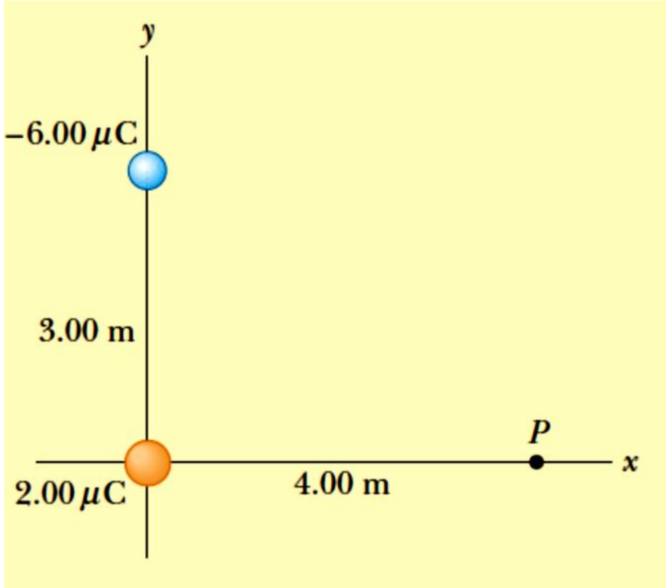
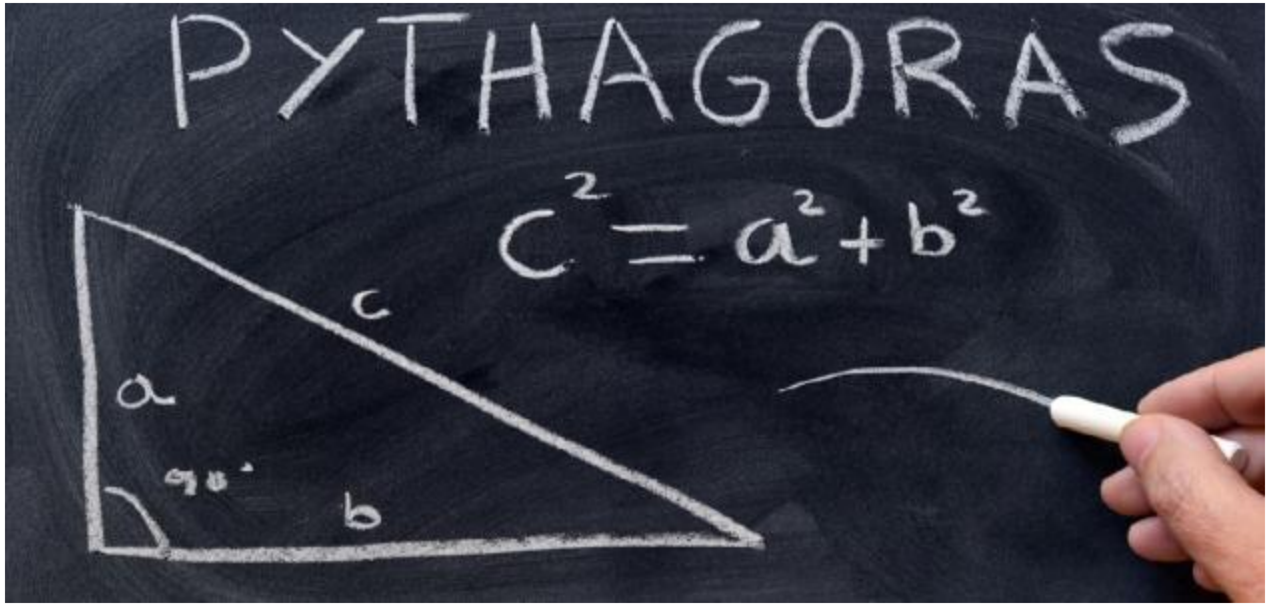


Fig. 6 (a)

3. Electric Potential and Potential Energy Due to Point Charges

Example 3: A charge q_1 **= 2.00 μC** is located at the origin, and a charge q_2 **= - 6.00 μC** is located at (0, 3.00) m, as shown in Figure 6a. **(a)** Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

Solution:



Q 1: Consider two charge as shows in Figure with the charge q_1= 3.0 nC is located at (0, 2.0) cm, and separation distance d=4.0 cm from another charge q_2= –3.0 nC. Find the total electric potential due to these charges at the point *P* whose coordinates are **(a)** (0, 1.0 cm), **(b)** (0, –5.0 cm)?

Solution:

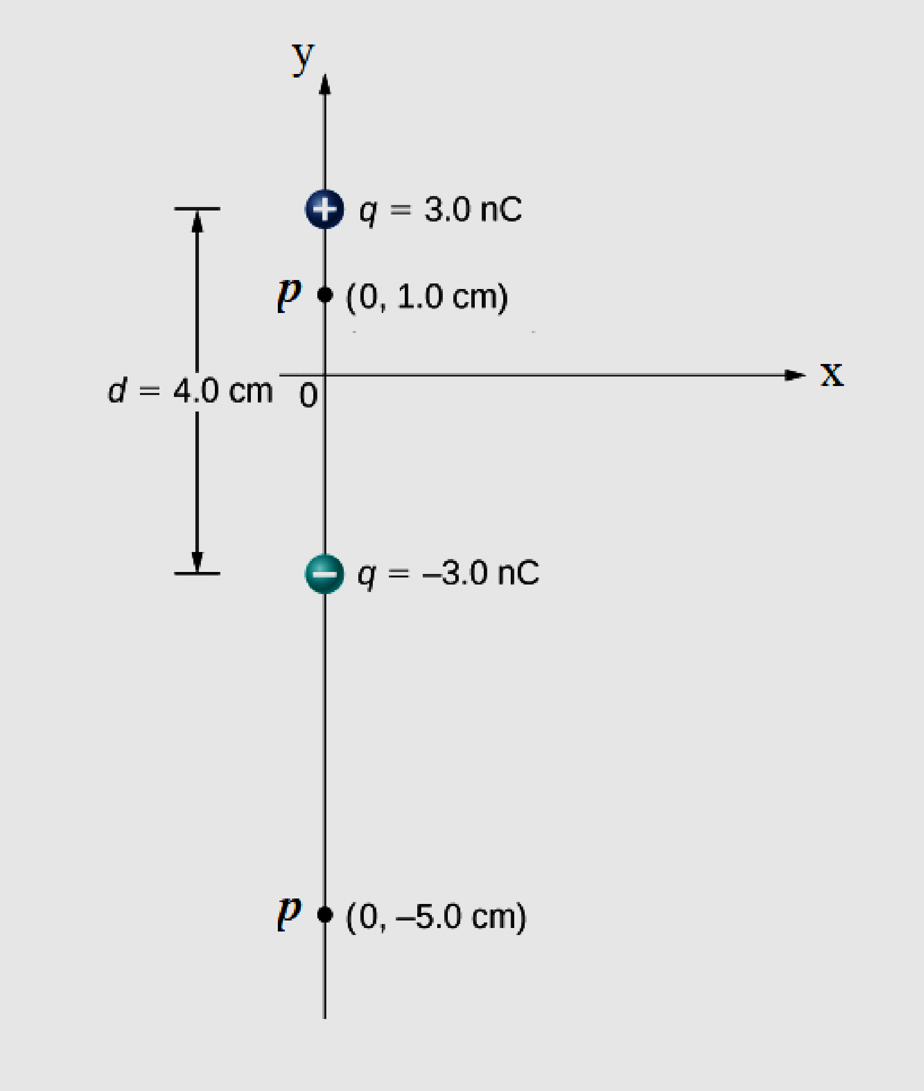
$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

(a)

$$V_P = 8.99\times10^{9}\,\frac{N.m^2}{C^2}\left(\frac{3\times10^{-9}\,C}{0.01\,m} - \frac{3\times10^{-9}\,C}{0.03\,m}\right) = (\qquad)\,V$$

(b)

$$V_P = 8.99\times10^{9}\,\frac{N.m^2}{C^2}\left(\frac{3\times10^{-9}\,C}{0.07\,m} - \frac{3\times10^{-9}\,C}{0.03\,m}\right) = (\qquad)\,V$$



3. Electric Potential and Potential Energy Due to Point Charges

(**c**) Find the potential energy of the system of three charges (Fig. 6b).

Solution:

$$U = K\left(\frac{q_1\,q_2}{r_{12}} + \frac{q_1\,q_3}{r_{13}} + \frac{q_2\,q_3}{r_{23}}\right)$$

$$U = \left(9\times10^{9}\right)\left(\frac{\left(2\times10^{-6}\right)\left(6\times10^{-6}\right)}{3} + \frac{\left(2\times10^{-6}\right)\left(3\times10^{-6}\right)}{4} + \frac{\left(-6\times10^{-6}\right)\left(3\times10^{-6}\right)}{5}\right)$$

$$U \approx \; -5.5\times10^{-2}\,\text{J}$$

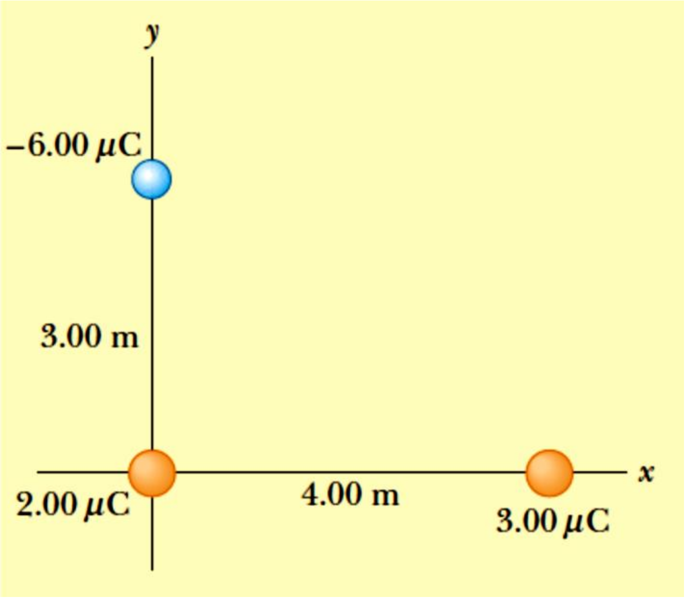
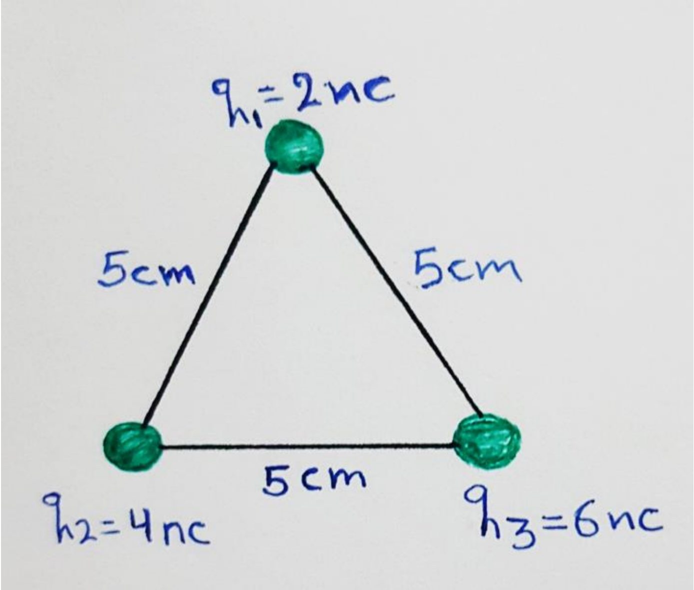


Fig. 6 (b)

Q 2: Find the potential energy of the system of three charges (q_1=2nC, q_2=4nC, q_3=6nC) moves from infinity to the corners of an equilateral triangle of side length 5 cm, where is the magnitude of electric potential due to these charges of 5.43×10^4 v?

Solution:



$$U = K\left(\frac{q_1\,q_2}{r_{12}} + \frac{q_1\,q_3}{r_{13}} + \frac{q_2\,q_3}{r_{23}}\right)$$

$$= 8.99 \times 10^9 \frac{N.m^2}{C^2}\left(\frac{(2\times10^{-9})(4\times10^{-9})}{0.05m} + \frac{(2\times10^{-9})(6\times10^{-9})}{0.05m} + \frac{(4\times10^{-9})(6\times10^{-9})}{0.05m}\right)$$

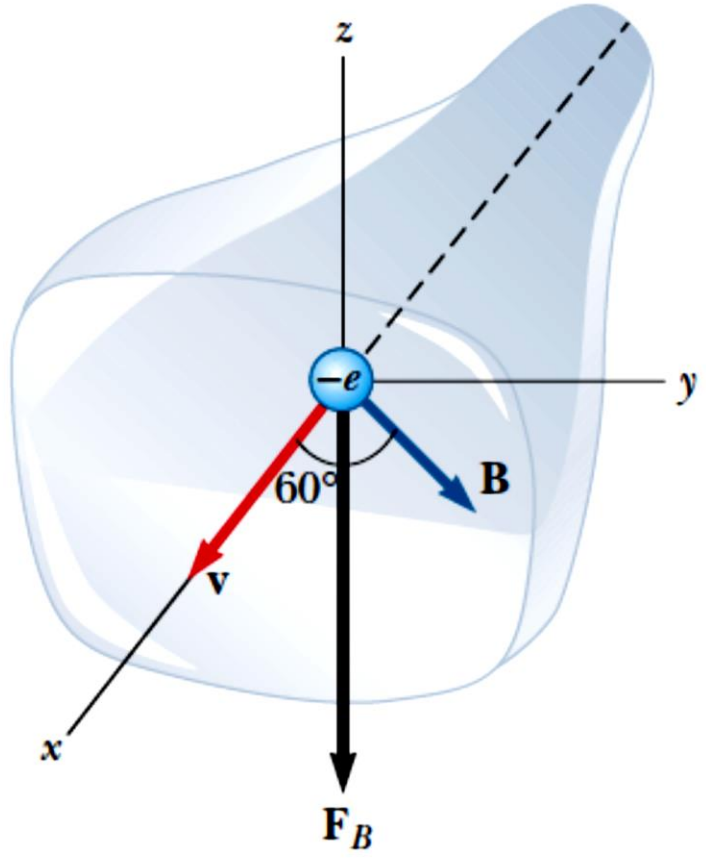
$$U = (\qquad\qquad)\,J$$

Example 1: An electron in a television picture tube moves toward the front of the tube with a speed of $\mathbf{8 \times 10^6}$ **m/s** along the x axis (Fig. 29.5). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude **0.025 T**, directed at an angle of **60°** to the x axis and lying in the **xy** plane. Calculate the magnetic force on and acceleration of the electron.

Solution:

$$F_B = |q|vB\sin\theta$$
$$= (1.6 \times 10^{-19}\ \text{C})(8.0 \times 10^{6}\ \text{m/s})(0.025\ \text{T})(\sin 60°)$$
$$= 2.8 \times 10^{-14}\ \text{N}$$

Because **v** × **B** is in the **positive** z direction (from the righthand rule) and the charge is **negative**, $\mathbf{F_B}$ is in the **negative** z direction.



Q 3: Calculate the magnitude of magnetic field when a proton moved perpendicularly (90^o) across a magnetic field with a speed of 7×10^5 m/s if the exert magnetic force on this proton of 2.5×10^{-12} N?

Solution:

$$F_B = |q|\, v_d\, B \sin\theta \quad (\sin\theta = 1)$$

$$\theta = 90^\circ, \quad \sin 90 = 1$$

$$F_B = q\, v_d\, B$$

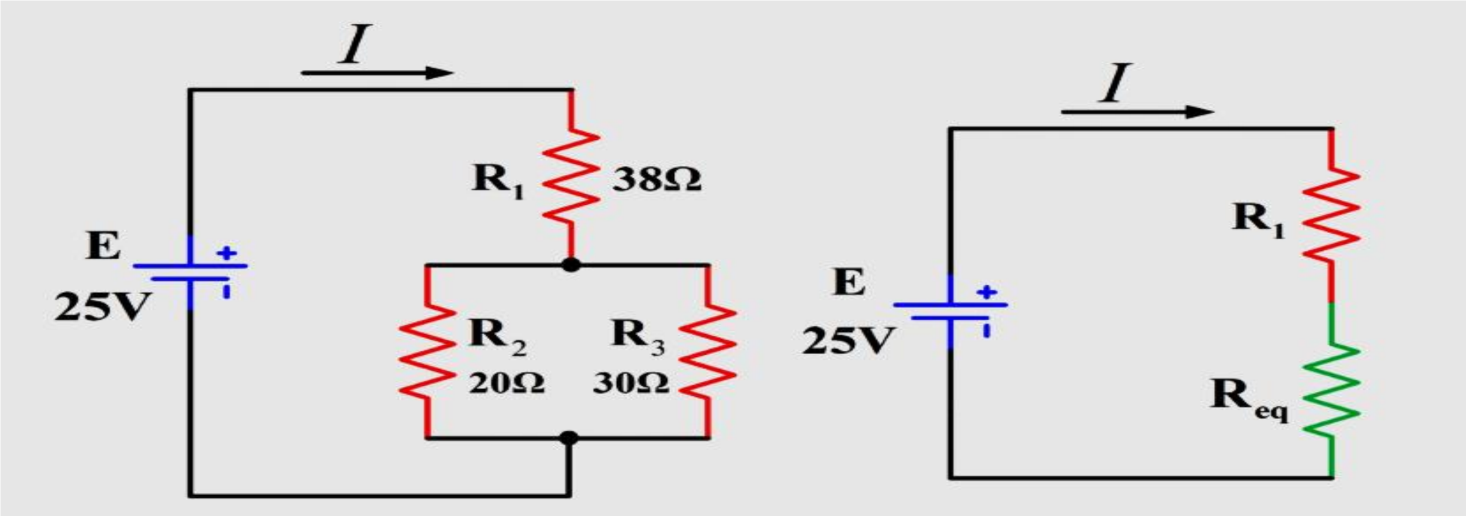
$$B = \frac{F_B}{q v} = \frac{2.5 \times 10^{-12}\ N}{(1.6 \times 10^{-19}\ C)(7 \times 10^5)} = (\qquad)\ \frac{N}{C \cdot m/s}$$

Q 4: Three resistors are connected in Series-Parallel as shown in (Figure a).

(1) Calculate the equivalent resistance of the circuit.

(2) Find the current drawn from the power supply in the circuit shown in (Figure b).

Solution:



(a) Series-Parallel Resistor Circuit (b) Equivalent Series Circuit

$$R_{eq_1} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{20\Omega} + \frac{1}{30\Omega} = \boxed{12\Omega}$$

$$R_{eq_2} = R_{eq_1} + R_3 = 12 + 38 = \boxed{50\Omega}$$

$$I = \frac{\Delta V}{R} = \frac{25V}{50\Omega} = \boxed{0.5A}$$

Example 26.2 The Cylindrical Capacitor

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$ (Fig. 26.5a). Find the capacitance of this cylindrical capacitor if its length is ℓ.

Solution It is difficult to apply physical arguments to this configuration, although we can reasonably expect the capacitance to be proportional to the cylinder length ℓ for the same reason that parallel-plate capacitance is proportional to plate area: Stored charges have more room in which to be distributed. If we assume that ℓ is much greater than a and b, we can neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them (Fig. 26.5b). We must first calculate the potential difference between the two cylinders, which is given in general by

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

where $\mathbf{E}$ is the electric field in the region $a < r < b$. In Chapter 24, we showed using Gauss's law that the magnitude of the electric field of a cylindrical charge distribution having linear charge density λ is $E_r = 2k_e\lambda/r$ (Eq. 24.7). The same result applies here because, according to Gauss's law, the charge on the outer cylinder does not contribute to the electric field inside it. Using this result and noting from Figure 26.5b that $\mathbf{E}$ is along r, we find that

$$V_b - V_a = -\int_a^b E_r\,dr = -2k_e\lambda\int_a^b \frac{dr}{r} = -2k_e\lambda\ln\left(\frac{b}{a}\right)$$

Substituting this result into Equation 26.1 and using the fact that $\lambda = Q/\ell$, we obtain

$$C = \frac{Q}{\Delta V} = \frac{Q}{\dfrac{2k_eQ}{\ell}\ln\left(\dfrac{b}{a}\right)} = \frac{\ell}{2k_e\ln\left(\dfrac{b}{a}\right)} \qquad \textbf{(26.4)}$$

where ΔV is the magnitude of the potential difference, given by $\Delta V = |V_b - V_a| = 2k_e\lambda\ln(b/a)$, a positive quantity. As predicted, the capacitance is proportional to the length of the cylinders. As we might expect, the capacitance also depends on the radii of the two cylindrical conductors. From Equation 26.4, we see that the capacitance per unit length of a combination of concentric cylindrical conductors is

$$\frac{C}{\ell} = \frac{1}{2k_e\ln\left(\dfrac{b}{a}\right)} \qquad \textbf{(26.5)}$$

An example of this type of geometric arrangement is a *coaxial cable*, which consists of two concentric cylindrical conductors separated by an insulator. The cable carries electrical signals in the inner and outer conductors. Such a geometry is especially useful for shielding the signals from any possible external influences.

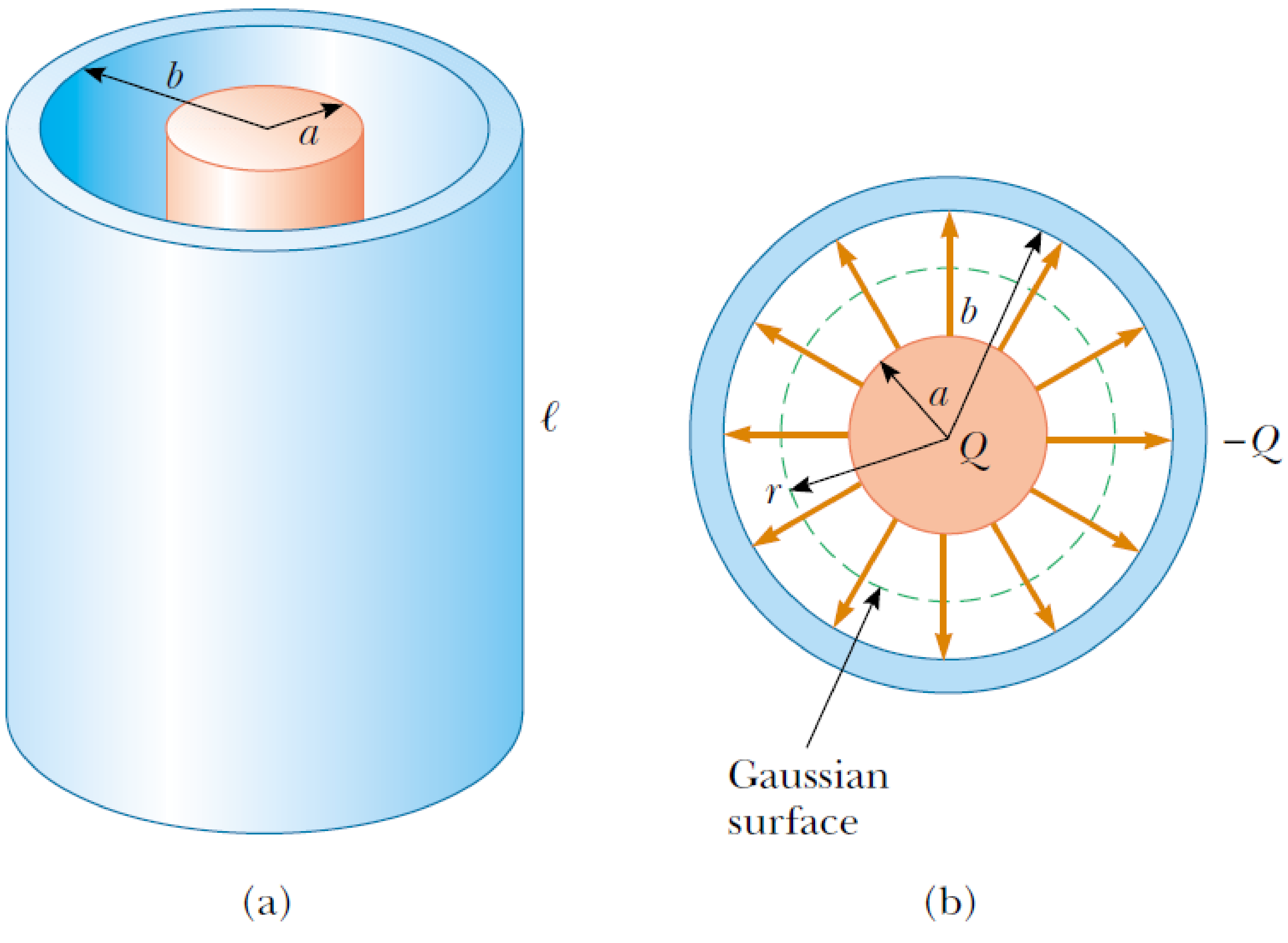


Figure 26.5 (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius a and length ℓ surrounded by a coaxial cylindrical shell of radius b. (b) End view. The dashed line represents the end of the cylindrical gaussian surface of radius r and length ℓ.