Chapter 1

Chapter Objectives:

- To describe different loads that subjected to building.
- To introduce student to different types of slabs in real life structures.
- To demonstrate the relation between slab and beam.
- To define drop panel, capital column and shear cap and how they different from each other's.
- To provide many examples for calculating slab thickness according to ACI Code.

1.1 Introduction

A reinforced concrete slab is a structural member, usually horizontal whose depth h, **is small** compared to their length, L and width, S. It may be supported by reinforced concrete beams, by masonry or reinforced concrete wall, by structural steel members, directly by columns.

The structural systems designed in third year (junior year) involved oneway slabs that carried load to beams, which, in turn, transmitted to column, and two-way slabs by coefficient method (**Method Three**), this method is no longer available in recent versions of ACI codes, for more about this method consult **Design of reinforced Concrete Structures by Nilson 1964**.

1.2 Loads

Loads that act on structures can be divided into three broad categories: **dead load**, **live load**, and **environmental loads**.

1.2.1 Dead Loads

- Are those that are constant in magnitude and fixed in location throughout the lifetime of the structure.
- Usually the major part of the dead load is the weight of the structure itself. This can be calculated with good accuracy from the design configuration, dimensions of the structure, and density of the material.
- For buildings, floor fill, finish floor, and plastered ceilings are usually included as dead loads.

1.2.2 Live Loads

Consist chiefly of **occupancy loads** in buildings and **traffic loads** on bridges. They may be either fully or partially in place or not present at all, and may also change in location. Their magnitude and distribution at any given time are uncertain, and even their maximum intensities throughout the lifetime of the structures are not known with precision for values for live load to be used in building are found in **ASCE 7-16**.



Figure 1.1: Live Loads.

1.2.3 Environmental Loads

Consist mainly of snow loads, wind pressure, earthquake loads, soil pressures on subsurface portions of structures, loads from possible ponding of rainwater on flat surface, and force caused by temperature differentials. Like live load s, environmental loads at any given time are uncertain in both magnitude and distribution.



1.3 Types of Slabs 1.3.1 One Way Slabs

Slab may be supported on **two opposite sides only** as shown in **Fig. 1.4**, in which case the structural action of the slab is essentially one-way, the loads being carried by the slab in the direction perpendicular to the supporting beams the design of one-way slab was discussed in junior year.



Fig.1.4: One-way slab supported on two opposite sides only.

There may be beams on all four sides, as shown in **Fig. 1.5** and of the ratio of length to width of one slab panel is larger than about 2, most of the load is carried in **short direction** to the supporting beams and **one-way action** is obtained in effect, **even though supports are provided on all sides**.



Fig.1.5: One-way slab supported on four sides with ratio of length to width larger than 2.

• For large column spacing, load may be transferred from the slab to the floor beams, then to larger beams (usually called the **girders**) and in turn to the supporting columns.



Fig.1.6: One-way slab with girders beams

1.3.2 Two Way Slab

1.3.2.1 Two Way Slab with Beams between Supports

When slab supported on four sides with ratio of length to width equal or less than 2, so that two-way slab action is obtained as shown in **Fig. 1.7**.



Fig.1.7: Two-way slabs supported on four sides with ratio of length to width equal or less than 2.

1.3.2.2 Two Way Slab without Beams between Supports 1.3.2.2.1 Flat Plate

Concrete slabs in some cases may be carried directly by columns, as shown in **Fig. 1.8**. Without the use of beams or girders, such slabs are described as **flat plates**, this type of slab maybe used when the spans are not large and loads particularly not heavy.



Plan View

Elevation View



Fig.1.8a: Two-way flat plates.



Fig.1.8b: Two-way flat plates.

1.3.2.2.2 Flat Slab

Flat slab is also beamless but incorporates a thickened slab region in the vicinity of the column and often employs flared column tops. Both are devices to reduce the **stresses due to negative bending and shear** around the columns, they are referred as **drop panels** and **column capitals** respectively. **Fig. 1.9a and b** is showing the flat slab.

Drop panel and capital column will be discussed briefly in articles **1.7** and **1.8** respectively.



Fig.1.9a: Two-way flat slabs with both drop panels and column capitals.



Fig.1.9b: Two-way flat slabs with both drop panels and column capitals.



Fig.1.9c: Two-way flat slabs with drop panels and column capitals in Baghdad.

Slabs may have only drop panels, this type of slab commonly used in parking garage as shown if **Fig. 1.10**.



Fig.1.10a: Two-way flat slabs with drop panels only.



Fig.1.10b: Two-way flat slabs with drop panels only.

Slab may be constructed with capital column **without** drop panel as shown in **Fig1.11**; this type of slab is rarely.



Fig.1.11b: Two-way flat slabs column capital only.

1.4 Relation between Slab and Beam

Very often slabs built **without interior beams** between the columns but the **edge beams** running around **the perimeter** of the building as shown in **Fig. 1.12**. These beams are very helpful in **stiffening** the slabs and **reducing the deflections** in the exterior slab panels.



Fig.1.12: Flat plate with edge beams.

• The stiffness of slabs with edge beams is expressed as a function of α_f . This expression is used to represent the ration of **flexural stiffness of a beam** ($\mathbf{E}_{cb} \times \mathbf{I}_b$) to flexural stiffness of the slab ($\mathbf{E}_{cs} \times \mathbf{I}_s$). If no beams are used as in the case for the flat plate α_f will equal to 0.

$$\alpha_{\rm f} = \frac{E_{\rm cb} \times I_{\rm b}}{E_{\rm cs} \times I_{\rm s}} = \frac{E_{\rm cb} \times I_{\rm b}}{E_{\rm cs} \times I_{\rm s}} = \frac{I_{\rm b}}{I_{\rm s}}$$

- In which E_{cb} and E_{cs} are the moduli of elasticity of the beam and slab concrete (usually the same) and I_b and I_s are the moments of inertia of the effective beam and the slab.
- The moment of inertia of a flange beam about it is own centroid axis can be computed based on simple definition of centroid or by approximate method many designers use the approximate method.

$$I_b \approx k \times \frac{b_w \cdot h^3}{12}$$
$$k = 1 + 0.2 \times \left(\frac{b_e}{b_w}\right)$$

• If both of $2 < \frac{b_e}{b_w} < 4$ and $0.2 < \frac{h_f}{h} < 0.5$ are satisfied then we can use the equation above.



L - Beam (for exterior beams only) T - Beam for (interior beam only) Fig.1.13: L and T beam shapes.

While I_s is the slab moment of inertia:

$$\mathbf{I}_{\rm s} = \frac{\boldsymbol{\ell}_{\rm s} \times \mathbf{h}_{\rm slab}^3}{12}$$

 $\boldsymbol{\ell}_{s}$ = is the width of the slab (frame) in the direction **perpendicular** to the beam that required to calculate α_{f} .

 $\mathbf{h}_{slab} = thickness of slab.$

• It's useful to remember the following notations:

h_f: is flange thickness and in our case is slab thickness.

b_w: with of web.

b_e: width of flange and sometimes it's notated by **b**_f.

h: total beam depth.



Example: Calculate I_b for following beams:



 $k = 1 + 0.2 \times \left(\frac{b_e}{b_w}\right) = 1 + 0.2 \times (3.67) = 1.734$ $\therefore I_b \approx k \times \frac{b_w \cdot h^3}{12} = 1.734 \times \frac{300 \times 600^3}{12} = 9.3636 \times 10^9 \,\text{mm}^4$

1.5 Types of Frame in Slab

Generally, there are two types of frames:
1. Longitudinal frame
o Exterior frame.
o Interior frame.
The longitudinal frames take abscissa axis (y – axis) always.

2. Transverse frame

Χ Longitudinal Direction

- Exterior frame.
- Interior frame.

The longitudinal frames take **ordinate** axis (x– axis) always.

▶ It's so important to know these types of frames, because we will see later that in chapter two, that these frames are the main concern in our problems.

Example: Calculate ℓ_s for interior and exterior frames, column dimensions are $(400 \times 400) \text{ mm}$





Solution:

1. For exterior transverse frame



$$\ell_{\rm s} = \frac{7000}{2} + \frac{400}{2} = 3500 + 200 = 3700 \,\rm{mm}$$

2. For interior transverse frame



$$\ell_{\rm s} = \frac{7000}{2} + \frac{7000}{2} = 3500 + 3500 = 7000 \,\rm{mm}$$

3. For exterior longitudinal frame



 $\ell_{\rm s} = \frac{6000}{2} + \frac{400}{2} = 3000 + 200 = 3200 \text{ mm}$

4. For interior longitudinal frame



$$\ell_{\rm s} = \frac{6000}{2} + \frac{6000}{2} = 3000 + 3000 = 6000 \,\,\rm{mm}\,\,\blacksquare$$

Section (8.10.3.2.3) of ACI Code states that if the transverse span of panels on either side of the centerline of supports varies, ℓ₂ shall be taken as the average of adjacent transverse spans.

 ℓ_2 for figure shown beside is:

$$\ell_2 = \left(\frac{3}{2} + \frac{3.5}{2}\right) = 3.25 \text{ m}$$



Example 1: for the slab shown in figure below calculate α_f for **B1, B2, B3 and B4**. All beams and columns dimensions are (600 × 300) mm (300 × 300) mm respectively, and slab thickness is (180) mm







$$I_{s} = \frac{\ell_{s} \times h_{slab}^{3}}{12} = \frac{2150 \times 180^{3}}{12} = 1.045 \times 10^{9} \text{ mm}^{4}$$
$$\alpha_{f \text{ for } B1} = \frac{I_{b}}{I_{s}} = \frac{7.992 \times 10^{9}}{1.045 \times 10^{9}} = 7.64$$

Calculating α_f for B2

Moment of inertia for **B2** is equal to moment of inertia for **B1** because they have same dimensions.

$$I_{b} = 7.992 \times 10^{9} \text{ mm}^{4}$$

$$\ell_{s} = \frac{6000}{2} + \frac{300}{2} = 3000 + 150 = 3150 \text{ mm}$$

$$150 \text{ mm}^{4}$$

$$I_{50} \text{ mm}^{4}$$

$$I_{50} \text{ mm}^{4}$$

$$I_{s} = \frac{\ell_{s} \times h_{slab}^{3}}{12} = \frac{3150 \times 180^{3}}{12} = 1.53 \times 10^{9} \text{ mm}^{4}$$

$$\alpha_{f \text{ for } B2} = \frac{I_{b}}{I_{s}} = \frac{7.992 \times 10^{9}}{1.53 \times 10^{9}} = 5.223$$

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Calculating a_f for B3



Calculating α_f for B4

Moment of inertia for **B4** is equal to moment of inertia for **B3** because they have same dimensions.

$$I_{b} = 9.504 \times 10^{9} \text{ mm}^{4}$$

$$\ell_{s} = \frac{4000}{2} + \frac{4000}{2} = 2000 + 2000 = 4000 \text{ m}$$

$$I_{s} = \frac{\ell_{s} \times h_{slab}^{3}}{12} = \frac{4000 \times 180^{3}}{12} = 1.944 \times 10^{9} \text{ mm}^{4}$$

$$\alpha_{f \text{ for } B4} = \frac{I_{b}}{I_{s}} = \frac{9.504 \times 10^{9}}{1.944 \times 10^{9}} = 4.89 \blacksquare$$

Finally:



• It will show later that calculating α is important for calculating suitable slab thickness for slab with interior beams.

Example 2: For the slab with beams shown below calculate α_f for **B1**, **B2**, **B3** and **B4**. All beams and columns dimensions are (600 × 400) mm (400 × 400) mm respectively, and slab thickness is (150) mm





Slab section for B2 in transverse interior frame Calculating α_f for B3

Beam section for B2 in transverse interior frame

 $\alpha_{f} = \frac{I_{b}}{I_{s}}$ $I_{b} = 1.026 \times 10^{10} \text{ mm}^{4} \text{ (same I_{b} for B1, because they are having same dimensions)}$ $\ell_{s} = \frac{4000}{2} + 200 = 2000 + 200 = 2200 \text{ mm}$ $I_{s} = \frac{\ell_{s} \times h_{slab}^{3}}{12} = \frac{2200 \times 150^{3}}{12} = 6.18 \times 10^{8} \text{ mm}^{4}$ $\alpha_{f \text{ for B3}} = \frac{I_{b}}{I_{s}} = \frac{1.026 \times 10^{10}}{6.18 \times 10^{8}} = 16.6$



Calculating *a_f* for B4

$$\alpha_{f} = \frac{I_{b}}{I_{s}}$$

$$I_{b} = 1.188 \times 10^{10} \text{ mm}^{4} \text{ same } I_{b} \text{ for B2}$$

$$\ell_{s} = 2000 + 2000 = 4000 \text{ mm}$$

$$I_{s} = \frac{\ell_{s} \times h_{slab}^{3}}{12} = \frac{4000 \times 150^{3}}{12} = 1.125 \times 10^{9} \text{ mm}^{4}$$

$$\alpha_{f \text{ for B4}} = \frac{I_{b}}{I_{s}} = \frac{1.188 \times 10^{10}}{1.125 \times 10^{9}} = 10.56 \quad \blacksquare$$
Finally:





For slabs with beams between columns along exterior edges α_f for the edge beams may not be < 0.8.



Fig.1.8: Two-way flat slabs with edge beams.

Example 3: Check whether if the slab shown in figure below is considered as a slab with or without edge beam according to ACI requirements. Edge beams dimensions are (600×250) mm and slab thickness is (150) mm, Column dimensions (250×250) mm



Solution:

According to footnote in Table 8.3.1.1 of ACI Code for slabs with beams between columns along exterior edge α_f for the edge beams shall not be less than 0.8, otherwise the slab will be considered as slab without edge beams.

$$\alpha_{f} = \frac{I_{b}}{I_{s}}$$

$$2 < \frac{b_{e}}{b_{w}} = \frac{700}{250} = 2.8 < 4$$

and
$$0.2 < \frac{h_f}{h} = \frac{150}{600} = 0.25 < 0.5$$

 $\therefore k = 1 + 0.2 \times \left(\frac{b_e}{b_w}\right) = 1 + 0.2 \times (2.8) = 1.56$
 $I_b = k \times \frac{b_w \cdot h^3}{12} = 1.56 \times \frac{250 \times 600^3}{12} = 7.02 \times 10^9 \text{ mm}^4$
 $\ell_s = 3000 + \frac{250}{2} = 3000 + 125 = 3125 \text{ mm}$
 $I_s = \frac{\ell_s \times h_{slab}^3}{12} = \frac{3125 \times 150^3}{12} = 8.78 \times 10^9 \text{ mm}^4$
 $\alpha_f = \frac{I_b}{I_s} = \frac{7.02 \times 10^8}{8.78 \times 10^9} = 7.995 > 0.8$

 \therefore The slab panels are considered with edge beams

1.6 Behavior of Two-Way Slab

Two-way slabs bend under load into dish-shaped surface, so there is bending in both principal directions. As a result, they must be reinforcement in both directions by layers of bars that are perpendicular to each other.

The ACI Code specifies two methods for designing two-way slabs. These are the **Direct Design Method** and the **Equivalent Frame Method**.



- The direct-design method is emphasized in this course because an understanding of the method is essential for understanding the concepts of two-way slab design. In addition, it is an excellent method of checking slab design calculations.
- It will show later that **chapter two** in our textbook is devoted for direct design method and its application.

1.7 Minimum Thickness for Drop panel

Flat plates present a possible problem in transferring the shear at the perimeter of the columns. In other words, there is a danger that the columns may punch¹ through the slabs as shown in **Fig 1.9**. Then the slab can be strengthened by thickening of the slabs around the columns (**drop panels**) as shown in **Fig 1.6**.



Fig 1.9 punching of two-way flat slab.

¹ Punching of slab will be discussed briefly in **chapter three** in our textbook.

They are provided for <u>three</u> main reasons:

- 1. The minimum thickness of slab required to limit deflections may be **reduced** by 10% if the slab has drop panels, the drop panel stiffens the slab in the region of highest moments and hence reduces the deflection.
- 2. A drop panel can be used to **reduce the amount of negative-moment** reinforcement required over a column in flat slab.
- 3. A drop panel gives additional slab depth at the column, thereby **increasing** the area of the critical **shear perimeter**.
- According to ACI Code 8.2.4 a drop panel shall be project below the slab at least one-fourth of the slab thickness.

And shall extend in each direction from centerline of support a distance not less than **one-sixth** the span length measured from **center to center** of supports (columns or walls) in that direction as shown in **Fig1.10**.



Fig 1.10: Dimensions of Drop panel.

- If drop panels do not satisfy the length requirements given in ACI Code 8.2.4 still can be used for added shear strength and are sometimes referred as shear capitals or shear caps (ACI Code 8.2.5).
- The only dimensional restriction on shear caps is that their horizontal projection from the face of the column must be greater or equal to their vertical projection below the slab. ACI Code (8.2.5).



1.8 Column Capitals

Occasionally, the top of a column will be flared outward as shown below, then this called a column capital and it located directly below the slab or a drop panel that is cast **monolithically** with the column.





- This is done to provide a larger shear perimeter at the column and to reduce the clear span ℓ_n , used in computing the moments.
- The ACI code requires that the capital concrete be placed at the same time as the slab concrete. As a result, the floor forming becomes considerably more complicated and expensive. For this reason, other alternatives, such as drop panels or shear reinforcement in the slab, should be considered before capitals are selected.
- The diameter or effective dimension of the capital is defined in ACI Code article 8.4.1.4 as that part of the capital lying within the largest right circular cone or pyramid with a 90-degree vertex that can be included within the outlines of the supporting column.



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Effective diameter of column capital.



4th Stage

Example 1: Check the dimensions of drop panel shown in figure below according to ACI code. Column dimension is (600×600) mm, slab thickness is (200) mm, drop panel thickness is (400) mm.



Solution:

• Check projected thickness for drop panel

According to **ACI Code 8.2.4** a drop panel shall be project below the slab at least one-fourth of the slab thickness.

$$h_{drop} = 200 \text{ mm}? \frac{h_{slab}}{4}$$
$$h_{drop} = 200 \text{ mm}? \frac{200}{4}$$

 $h_{drop} = 200 \text{ mm} > 50 \text{ mm}$ the projected thickness is ok.

• Check length of drop panel

ACI Code state that each direction from centerline of support shall extend a distance not less than one-sixth the span length measured from center to center of supports in that direction

1.3? $\frac{\ell}{6}$ 1.3? $\frac{5}{6}$ 1.3 m > 0.83 m ok All requirements are satisfying for a drop panel ■ **Example 2:** Check the dimensions of drop panel shown in figure below according to ACI code. Column dimension is (600×600) mm, slab thickness is (200) mm, drop panel thickness is (500) mm.



Solution:

• Check the projected thickness for drop panel

According to **ACI Code 8.2.4** a drop panel shall be project below the slab at least one-fourth of the slab thickness.

$$h_{drop} = 300 \text{ mm}? \frac{h_{slab}}{4}$$
$$h_{drop} = 300 \text{ mm}? \frac{200}{4}$$

 $h_{drop} = 300 \text{ mm} > 50 \text{ mm}$ the projected thickness is ok.

• Check the length of drop panel

ACI Code state that each direction from centerline of support shall extend a distance not less than one-sixth the span length measured from center to center of supports in that direction

$$0.6? \frac{\ell}{6}$$

 $0.6? \frac{5}{6}$

0.6 m < 0.83 m not ok, dimensions need to be increasing \blacksquare

1.9 Minimum Thickness for Two Way Slab

1.9.1 Minimum Thickness for Flat and Plate Slabs.

To ensure that slab deflections is service will not be troublesome, the best approach is to compute deflections with limiting value, methods have been developed that are both simple and acceptable accurate of predicting deflection of two-way slabs.

Alternatively, deflection control can be achieved indirectly by adhering to more or less arbitrary limitation of minimum slab thickness, limitations developed from **review of test** data and **study of the observed** deflections of **actual structures**.

ACI Code Table 8.3.1.1 establishes minimum thickness for two-way slabs; simplified criteria are included pertaining slab thickness for flat and plate slabs (without interior beams).

	Without drop panels ^[3]			With drop panels ^[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
fy, MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]	
280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	ℓ _n /36	<i>ℓ</i> _n /40	ℓ _n /40
420	ℓ _n /30	ℓ _n /33	ℓ _n /33	ℓ _n /33	<i>ℓ</i> _n /36	ℓ _n /36
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	ℓ _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

 ${}^{[1]}\ell_n$ is the clear span in the long direction, measured face-to-face of supports (mm).

^[2]For f_y between the values given in the table, minimum thickness shall be calculated by linear interpolation.

^[3]Drop panels as given in 8.2.4.

^[4]Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if α_f is less than 0.8. The value of α_f for the edge beam shall be calculated in accordance with 8.10.2.7.

In all cases, the minimum thickness of slabs without interior beams **must not less** than the following:

For slabs withoutdrop panels:125 mmFor slabs withdrop panels:100 mm

 ℓ_n = is the length of the clear span in **the long direction** of two-way slab, measured **face to face** of the **supports** (column or capital column) for slabs without beams. See the **Fig.1.11**.



Fig. 1.11a clear span in case of drop panel and capital column.

Fig. 1.11b clear span in case of drop panel only.



Fig. 1.11c clear span in case of column. Fig. 1.11d clear span in case of capital column.

- Someone may ask, why ACI Code in table 8.3.1.1 suggests larger slab thickness for exterior panel than for interior panel.
 - To proof this aspect of code, let's consider we have flat plate with three continuous panels with equal spans lengths in each direction.



• By using any finite element software, in this example we used **SAFE** to calculate deflections for both interior and exterior panels, and we found that deflection for exterior panels is larger than interior panels, to limit this deflection we must increase slab thickness for exterior panel.



- In interior panel moment distribution will be taking place from four sides, therefore deflection will be reduced, therefore smaller slab thickness will be needed for interior panel when we compare it with exterior panels.
- And as was mentioned before, both of edge beams and drop panels try to stiffen slab and this reduce deflection, therefore smaller slab thickness will be required if edge beams or drop panel exist.

1.9.2 Types of Panels

Panels can be classified according to their **locations** into **interior** and **exterior** panels. Figure below shows the type of panels according to their locations.



• Interior panel must be continuous by **four** sides





Solution:

For exterior panel

 $\ell_n = 5000 - 150 \times 2 = 4700 \text{ mm}$

By using ACI code Table 8.3.1.1

Table 8.3.1.1—Minimum thickness of nonp	re-
stressed two-way slabs without interior be	ams
(mm) ^[1]	

	Without drop panels ^[3]			With drop panels ^[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
<i>f</i> _y , MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]	
280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	<i>ℓ</i> _n /40	<i>ℓ</i> _n /40
420	• ℓ _n /30	ℓ _n /33	ℓ _n /33	ℓ _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36
520	ℓ _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34

h =
$$\frac{\ell_n}{30} = \frac{4700}{30} = 156 \text{ mm} > 125 \text{ O.K}$$

h $\approx 160 \text{ mm}$

For interior panel

	Without drop panels ^[3]			With drop panels ^[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
fy, MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]	
280	ℓ _n /33	ℓ _n /36	$\ell_n/36$	ℓ _n /36	<i>ℓ</i> _n /40	ℓ _n /40
420	l _n /30	ℓ _n /33-►	ℓ _n /33	ℓ _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

h = $\frac{\ell_n}{33} = \frac{4700}{33} = 142 \text{ mm} > 125 \text{ mm O.K}$ h $\approx 150 \text{ mm.}$

• In practice the structural engineer uses same slab thickness for all panels.

Therefore, the suitable slab thickness to control deflection = 160 mm

Example 2: Resolve the previous example by assuming $f_y = 350$ MPa **Solution:**

: $f_y = 350$ MPa doesn't exist in **ACI Table 8.3.1.1**, therefore interpolation shall be taking place.

For exterior panel





$$\frac{36-33}{420-280} = \frac{X}{420-350}$$

X =1.5
Total factor = 33 + 1.5 = 34.5
Or (36 + 303/2 = 34.5 (because of 350 lies in **mid** between 420 and 280)
 $h = \frac{4700}{34.5} = 136 \text{ mm} > 125 \text{ mm O.K}$
 $h \approx 140 \text{mm}$
For all panels use the larger h = 150 mm \blacksquare

Example 2: Check the slab thickness and drop panel dimensions according to ACI Code for building indicated in Fig. below, use $f_y = 420$ MPa, slab thickness is 200 mm, drop panel thickness is 420 mm.



Solution:

• Check drop panel dimesntions

 $\frac{2500}{2} ? \frac{\ell_1}{6}$ $1250 \text{ mm} > \frac{4000}{6} = 666.66 \text{ mm O.K}$ $\frac{2500}{2} ? \frac{\ell_1}{6} = \frac{5000}{6} = 833.33 \text{ mm}$ $1250 \text{ mm} > \frac{5000}{6} = 833.33 \text{ mm}$

: Dimensions of drop panel are satisfied ACI Code requirements.

• Check drop panel thickness

According to **ACI Code 8.2.4** the project part of drop panel shall be below the slab at least **one-fourth** of the **slab thickness**.



$$220? \frac{\text{hslab}}{4} \\ 220 > \frac{200}{4} = 50 \text{ mm}$$

: Thicknesses of drop panel are satisfied ACI Code requirement.

• Checking Slab thickness For exterior panel

$$\ell_n = 5000 - 150 \times 2 = 4700 \text{mm}$$

 $h = \frac{\ell_n}{33} = \frac{4700}{33} = 142 \text{ mm} > 100 \text{ mm}$
 $h \approx 150 \text{ mm}$

200mm > 150 mm slab thickness is O.K

For interior panel

$$\ell_{\rm n} = 5000 - 150 \times 2 = 4700 {\rm mm}$$

$$h_{\min} = \frac{\ell_n}{36} = \frac{4700}{36} = 130.5 \text{mm} > 100 \text{ mm O.K}$$

 $h \approx 140 \text{ mm}$

200 mm > 140 mm the thickness is O.K. \blacksquare

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]



Table 8.3.1.1—Minimum	thickness of nonpre-
stressed two-way slabs	without interior beams
(mm) ^[1]	

	Witho	Without drop panels ^[3]			ı drop pan	iels ^[3]		
			Interior	Interior		Interior		
	Exterio	Exterior panels		Exterio	r panels	panels		
	Without	With		Without	With			
f_y ,	edge	edge		edge	edge			
MPa ^[2]	beams	beams ^[4]		beams	beams ^[4]			
280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	ℓ _n /36	ℓ _n /36	ℓ _n /40	$\ell_n = 0$		
(420)	n/0 0	"/# 3 "	u n/ u 3 u	■ ■ _n /∎3 ■		<i>l</i> _n /36		
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	ℓ _n /31	ℓ _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34		

• According to ACI Code 8.10.1.3 circular or polygon-shaped supports shall be treated as a square support with same area.



Example 4: The architectural engineer is proposed 200 mm slab thickness, check weather if this thickness is satisfying the deflection requirement of ACI Code, use $f_y = 420$ MPa.



Example 5: find slab thickness due to deflection requirements, flat slab shown in figure beside with edge beams Column and capital diameter = 1000 mm ($f_y = 420 \text{ gm MPa}$)

<u>Solution</u> For Exterior Panel

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

	Without drop panels ^[3]			With drop panels ^[3]		
			Interior			Interior
	Exterio	r panels	panels	Exterio	r panels	panels
	Without	With		Without	With	
f_{y}	edge	edge		edge	edge	
MPa ^[2]	beams	beams ^[4]		beams	beams ^[4]	
280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	ℓ _n /36	ℓ _n /36	ℓ _n /40	ℓ _n /40
420	<u>ℓ</u> _n /30	<i>ℓ</i> _n /33	ℓ _n /33	ℓ _n /33	<i>ℓ</i> _n /36	ℓ _n /36
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34

$$h = \frac{\ell_n}{33}$$

$$\ell_n = 8000 - 0.89 \times 500 \times 2 = 7110 \text{ mm}$$

$$h = \frac{7110}{33} = 215.4 \text{ mm} > 125 \text{ mm O.K}$$

$$h \approx 220 \text{ mm}$$

For Interior Panel

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

	Without drop panels ^[3]			With drop panels ^[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
fy, MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]	
280	ℓ _n /33	<i>ℓ</i> _n /36	ℓ _n /36	ℓ _n /36	ℓ _n /40	ℓ _n /40
420	<u> l_u/30</u>	<u>ℓ_n/33</u>	ℓ _n /33	ℓ _n /33	<i>ℓ</i> _n /36	ℓ _n /36
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	ℓ _n /34

 $h = \frac{7110}{33} = 215.4 \text{ mm} > 125 \text{ mm O.K}$ h ≈ 220 mm ■ ∴ Use h = 220 mm





Typical section for capital column.

Example 6: find slab thickness due to $_$ deflection requirements, flat slab shown in figure beside without edge beams Column and capital diameter = 1000 mm (f_y = 420 $\stackrel{\boxtimes}{\sim}$ MPa)

Solution For exterior panel

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

	Without drop panels ^[3]			With drop panels ^[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
fy, MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]	
280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	ℓ _n /36	<i>ℓ</i> _n /36	ℓ _n /40	<i>ℓ</i> _n /40
420 -	ℓ _n /30	<i>ℓ</i> _n /33	ℓ _n /33	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	ℓ _n /34

$$h = \frac{\ell_n}{30}$$

$$\ell_n = 8000 - 0.89 \times 500 \times 2 = 7110 \text{ mm}$$

$$h = \frac{7110}{30} = 237 \text{ mm} > 125 \text{ mm O.K}$$

$$h \approx 240 \text{ mm}$$

For interior panel

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams $(mm)^{[1]}$

	Without drop panels ^[3]			With drop panels ^[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f _y , MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]	
280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	ℓ _n /40	<i>ℓ</i> _n /40
420	<u>ℓ</u> _n /30	<u>ℓn/33</u>	ℓ _n /33	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	ℓ _n /36
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34

$$h = \frac{\ell_n}{\frac{33}{33}}$$

$$h = \frac{7110}{\frac{33}{33}} = 215.4 \text{ mm} > 125 \text{ mm O.K}$$

$$h \approx 220 \text{ mm}$$

∴ Use h = 240 mm ■



Example 7: Check the slab thickness according to deflection requirement if the slab thickness is (200) mm, and the slab is supported by edge beam (300×600) mm f_y = 420 MPa.



Solution:

The slab is with edge beam.

 $\ell_{\rm n} = 6000 - 2 \times 150 = 5700 \,\rm mm$

For exterior panel

h = $\frac{\ell_n}{33}$ = $\frac{5700}{33}$ = 172.73mm > 125 mm O.K h ≈ 180 mm 200 mm > 180 mm slab thickness is O.K ■

Example 8: Check the slab thickness according to deflection requirement for example 5 by considering there are no edge beams, the slab thickness proposed by structural engineer is 150 mm, $f_y = 420$ MPa

Solution:

For exterior panel

 $\ell_{\rm n} = 6000 - 2 \times 150 = 5700 \, \rm mm$

 $h = \frac{\ell_n}{30} = \frac{5700}{30} = 190 \text{ mm} > 125 \text{ mm O.K}$

150 mm < 190 mm slab thickness is not O.K

 \therefore Slab thickness must be increasing to 190 mm or larger

<u>Homework 1:</u> Find the minimum required slab thickness according to ACI Code for the slab shown below Fig. use $f_y = 420$ MPa.



Homework 2: Find the minimum required slab thickness according to ACI Code for the slab shown below Fig. use $f_y = 420$ MPa. Column dimensions are (300×300) mm, and beam dimensions are (300×600) mm.



4th Stage

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

1.5.2 Minimum Thickness for Two-Way Slab with Interior Beams

The parameter used to define the relative stiffness of the beam and slab spanning

 $\alpha_{\rm f} = \frac{E_{\rm cb} \times I_{\rm b}}{E_{\rm cc} \times I_{\rm s}} = \frac{I_{\rm b}}{I_{\rm s}}$

According to ACI code, the two-way slab with interior beams can be

Minimum h, mm

8.3.1.1 applies

(a)

in either direction is α_{f} it can be calculated by equation below:

0.2 ≤ α _{fm} ≤ 2.0	Greater of:	$\frac{\alpha(1400)}{36+5\beta(\alpha_{fm}-0.2)}$	(b) ^{[2],[3]}
		125	(c)
α _{fm} > 2.0	Greater of:	$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$	(d) ^{[2],[3]}
		90	(e)
^[1] α _{fm} is the average τ	alue of α _f for	all beams on edges of a panel and α_f s	hall be calcu-
lated in accordance v	with 8.10.2.7.		

 ${}^{[2]}\ell_n$ is the clear span in the long direction, measured face-to-face of beams (mm).

[3]β is the ratio of clear spans in long to short directions of slab.

Then α_{fm} is defined as **the average** value of α_f **for all the beams** on the **edges** of a given panel.

 ℓ_n = is the length of the clear span in **the long direction** of two-way slab, measured face to face of the beams. see Fig. 1.12.



Fig. 1.12 clear span in case of interior beams.

It was discussed briefly in article 1.5.

 $\alpha_{fm}^{[1]}$

 $\alpha_{fm} \leq 0.2$

Example 1: for the slab with interior beams shown in figure below, find the minimum thickness for slab due to deflection requirements, α_f for each beam are indicated on figure, use $f_y = 420$ MPa, column dimensions are (400×400) mm



Solution:

For panel 1 $\alpha_{\rm fm} = \frac{7.64 + 3.26 + 4.89 + 3.26}{4} = 4.763 > 2$

from ACI Code table 8.3.1.2

Table 8.3.1.2—Minimum thickness of nonpre-stressed two-way slabs with beams spanningbetween supports on all sides

α _{fm} [1]			
$\alpha_{fm} \leq 0.2$		(a)	
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta \left(\alpha_{,fm} - 0.2\right)}$	(b) ^{[2],[3]}
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$	(d) ^{[2],[3]}
		90	(e)

$$h = 1.1 \times \frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36+9\beta}, \beta = \frac{\text{clear span in long direction}}{\text{clear span in short direction}} = \frac{6000 - 400}{4000 - 400} = \frac{5600}{3600} = 1.56$$



h = 1.1 ×
$$\frac{5600 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.56}$$
 123.1 mm > 90 mm O.K
∴ h ≈ 130 mm ■
For panel 2
 $\alpha_{fm} = \frac{7.64 + 3.26 + 4.89 + 5.22}{4} = 5.252 > 2$
h = 1.1 × $\frac{5600 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.56}$ 123.1 mm > 90 mm O.K
∴ h ≈ 130 mm ■
For panel 3
 $\alpha_{fm} = \frac{4.89 + 3.26 + 4.89 + 5.22}{4} = 4.565 > 2$
h = 1.1 × $\frac{5600 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.56}$ 123.1 mm > 90 mm O.K
∴ h ≈ 130 mm ■

 $h = 1.1 \times \frac{\frac{5600 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.56}}{123.1} \text{ mm} > 90 \text{ mm O.K}$

For panel 4

 \therefore h \approx 130 mm

 $\alpha_{fm} = \frac{4.89 + 3.26 + 4.89 + 3.26}{4} = 4.075 > 2$

Example 2: Find slab thickness for the following panel, α_f for each beam are indicated in Fig. below, $f_y = 420$. Beam dimensions are (400 × 600) mm.

Solution:

Find $\alpha_{fm} = \frac{1 + 1.1 + 2 + 1.2}{4} = 1.325$ $0.2 < \alpha_{fm} = 1.325 < 2$



By using the equation in ACI Code Table 8.3.1.2:

h = $\frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{0.000}$ or 125 mm
$11 - 36 + 5\beta \times (\alpha_{\rm fm} - 0.2)$ 01 12.5 mm
$\beta = \frac{\ell_n}{\ell_n} = \frac{8000 - 400}{\ell_n} = \frac{7600}{\ell_n} = 1.357$
$P = s_n = 7000 - 400 = 5600$
$7600 \times \left(0.8 + \frac{420}{1400}\right)$
$\Pi = \frac{1}{36 + 5 \times 1.357 \times (1.325 - 0.2)}$
h = 196.6 mm > 125 mm O.K
$h \approx 200 \text{ mm}$

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides



Example 3: find suitable slab thickness for the interior panel that showing in figure below, α_f for each beam are 0.1, use $f_y = 420$ MPa and column dimensions are (300×300) mm



Solution:

Find $\alpha_{\rm fm} = \frac{0.1 + 0.1 + 0.1 + 0.1}{4} = 0.1$

 $\alpha_{fm}=0.1<0.2$ then ACI Table 8.3.1.1 shall be applied.

Table 8.3.1.2—Minimum thickness of nonpre-
stressed two-way slabs with beams spanning
between supports on all sides

α _{fm} [1]			
$\alpha_{fm} \leq 0.2$		8.3.1.1 applies	(a)
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta \left(\alpha_{fm} - 0.2\right)}$	(b) ^{[2],[3]}
		125	(c)
α _{fm} > 2.0	Greater of:	$\frac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+9\beta}$	(d) ^{[2],[3]}
		(e)	

 $\ell_n = 7000 - 300 = 6700 \text{ mm}$ h = $\frac{\ell_n}{33} = \frac{6700}{33} = 203 \text{ mm} > 125 \text{ mm O.K}$ h ≈ 210 mm ■

Table 8.3.1.1 - Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

	Witho	ut drop pa	anels ^[3]	With drop panels ^[3]			
	Exterio	r panels	Interior panels	Exterio	Exterior panels		
f _y , MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]		
280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	$\ell_n/36$	<i>ℓ</i> _n /36	ℓ _n /40	<i>ℓ</i> _n /40	
420	<mark>6</mark> n/-30	- € _n /33-►	<i>ℓ</i> _n /33	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34	

<u>Note:</u> According to ACI Code 8.3.1.2.1 at discontinuous edges of slabs, if an edge beam with $\alpha_f < 0.8$ then the minimum thickness required by (b) or (d) of Table 8.3.1.2 shall be increased at least 10 percent in the panel with discontinuous edge.



By using equation in ACI Code Table (8.3.1.2):

$$h = \frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \text{ or } q$$

or 90 (greater)

Table 8.3.1.2—Minimum thickness of nonpre-stressed two-way slabs with beams spanningbetween supports on all sides

α _{fm} [1]					
$\alpha_{fm} \leq 0.2$		8.3.1.1 applies	(a)		
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta \left(\alpha_{fm} - 0.2\right)}$	(b) ^{[2],[3]}		
		125			
$\alpha_{fm} > 2.0$	Greater of:	$\frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$	(d) ^{[2],[3]}		
		90	(e)		

Since α_f for edge beam 0.5 < 0.8, then the equation shall be increased by at least 10 percent.

$$h = 1.1 \times \frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36+9\beta}, \beta = \frac{\text{clear span in long direction}}{\text{clear span in short direction}} = \frac{8000 - 400}{6000 - 400} = \frac{7600}{5600} = 1.36$$
$$h = 1.1 \times \frac{7600 \times \left(0.8 + \frac{420}{1400}\right)}{36+9 \times 1.36} = 190.63 \text{ mm} > 90 \text{ mm O.K}$$

 \therefore h \approx 200 mm \blacksquare

<u>Homework</u>: Find slab thickness for the panel 1, α_f for each beam are indicated in Fig. below, $f_y = 420$ MPa. Beam dimensions are (300×600) mm.



1.6 General Examples for Calculating Two Way Slab Thickness

Example 1: Find the minimum thickness of a slab for an interior panel due to deflection control for the following: Use $f_y = 420$ MPa (60000 psi).

- a. Slab with beams (8.2 \times 7.7) m clear span with α_{fm} = 2.3
- b. Slab without drop panels (5.4 \times 4.8) m clear span with $\alpha_{fm} = 0.18$
- c. Flat plate (4.2×4.6) m clear span.
- d. Flat slab with drop panels (6×6.2) m clear span.
- e. Slab with beams (5.8 \times 5.8) m clear span with α_{fm} =1.5

Solution:

a. Slab with beams (8.2 \times 7.7) m clear span with $\alpha_{fm}{=}2.3$

: $\alpha_{fm}=2.3>2$ then ACI Table 8.3.1.2 shall be used

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides



h =
$$\frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$
 or 90 mm
 $\beta = \frac{\ell_n}{s_n} = \frac{8200}{7700} = 1.065$
h = $\frac{8200 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.065} = 197.87$ mm > 90 mm O.K.
∴ h ≈ 200 mm¹

¹ all slab thickness will be round up to nearest 10 mm.

b. Slab without drop panels (5.4 \times 4.8) m clear span with α_{fm} = 0.18

: α_{fm} = 0.18 < 0.2 then ACI Table 8.3.1.1 shall be used



h = $\frac{\ell_n}{33}$ = $\frac{5400}{33}$ = 163.65 mm > 125 mm O.K. h ≈ 170 mm ■

c. Flat plate (4.2×4.6) m clear span.

As there is nothing mentioned about α_{fm} then **ACI Table 8.3.1.1** shall be used directly.

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

	Witho	ut drop pa	anels ^[3]	With drop panels ^[3]			
	Exterio	r panels	Interior panels	Exterio	Interior panels		
fy, MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]		
280	ℓ _n /33	<i>ℓ</i> _n /36	ℓ _n /36	ℓ _n /36	ℓ _n /40	ℓ _n /40	
420	ℓ _n /30	<i>l</i> n/33- •	ℓ _n /33	ℓ _n /33	<i>ℓ</i> _n /36	ℓ _n /36	
520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	ℓ _n /31	ℓ _n /31	<i>ℓ</i> _n /34	ℓ _n /34	

h =
$$\frac{\ell_n}{33}$$
 = $\frac{4600}{33}$ = 139.4 mm > 125 mm O.K.
h ≈ 140 mm ■

d. Flat slab with drop panels (6 \times 6.2) m clear span.

As there is nothing mentioned about α_{fm} then **ACI Table 8.3.1.1** shall be used directly.

()								
	Witho	ut drop pa	anels ^[3]	With drop panels ^[3]				
	Exterio	r panels	Interior panels	Exterio	r panels	Interior panels		
fy, MPa ^[2]	Without edge beams	With edge beams ^[4]	-	Without edge beams	With edge beams ^[4]			
280	ℓ _n /33	ℓ _n /36	<i>ℓ</i> _n /36	ℓ _n /36	ℓ _n /40	<i>ℓ</i> _n /40		
420	<u>ℓ_n/30</u>	<u> l_n/33</u>	<u> </u>	<u>ℓn/33</u>	$\ell_n/36$	<i>ℓ</i> _n /36		
520	ℓ _n /28	<i>ℓ</i> _n /31	ℓ _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34		

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)^[1]

h = $\frac{\ell_n}{36} = \frac{6200}{36} = 172.2 \text{ mm} > 100 \text{ mm O.K.}$ h ≈ 175 mm ■

e. Slab with beams (5.8 × 5.8) m clear span with $\alpha_{fm} = 1.5$:: 0.2 < $\alpha_{fm} = 1.5 < 2$ then ACI Table 8.3.1.2 shall be used

Table 8.3.1.2—Minimum thickness of nonpre-stressed two-way slabs with beams spanningbetween supports on all sides



h =
$$\frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta \times (\alpha_{fm} - 0.2)}$$
 or 125 mm
 $\beta = \frac{\ell_n}{s_n} = \frac{5800}{5800} = 1$
h = $\frac{5800 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 5 \times 1 \times (1.5 - 0.2)} = 150.12$ mm > 125 mm O.K.
∴ h ≈ 160 mm

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Example 2: Find the minimum thickness of a slab for an interior panel due to deflection control for the following: Use $f_y = 420$ MPa (60000 psi)

- a. Flat slab with drop panels (6.4×6.4) m clear span
- b. Slab with beams (6.0 \times 6.0) m clear span with $\alpha_{fm} = 2.7$
- c. Slab with beams (5.4 \times 4.5) m clear span with $\alpha_{fm} = 0.7$.
- d. Flat plate (4×4) m clear span.

Solution:

a. Flat slab with drop panels (6.4 \times 6.4) m clear span From ACI Table 8.3.1.1

h =
$$\frac{\ell_n}{36}$$
 = $\frac{6400}{36}$ = 177.78 mm > 100 mm O.K.
∴ h ≈ 180 mm ■

- b. Slab with beams (6.0 \times 6.0) m clear span with α_{fm} =2.7
 - : $\alpha_{fm}=2.3>2$ then ACI Table 8.3.1.2 shall be used

h =
$$\frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$
 or 90 mm
 $\beta = \frac{\ell_n}{s_n} = \frac{6000}{6000} = 1$
h = $\frac{6000 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1} = 146.77$ mm > 90 mm O.K.
∴ h ≈ 150 mm ■

c. Slab with beams (5.4 \times 4.5) m clear span with α_{fm} =0.7.

$$\therefore 0.2 < \alpha_{\rm fm} = 0.7 < 2 \text{ then ACI Table 8.3.1.2 shall be used}$$

$$h = \frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta \times (\alpha_{\rm fm} - 0.2)} \text{ or } 125 \text{ mm}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{5400}{4500} = 1.2$$

$$h = \frac{5400 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 5 \times 1.2 \times (0.7 - 0.2)} = 152.3 \text{ mm} > 125 \text{ mm O.K.}$$

∴ h ≈ 160 mm ∎

d. Flat plate (4×4) m clear span.

From ACI Table 8.3.1.1

h =
$$\frac{\ell_n}{33}$$
 = $\frac{4000}{33}$ = 121.2 mm < 125 mm not O.K.
Use h = 125 mm ■

Example 3: Find the minimum thickness of a slab for an interior panel due to deflection control for the following: Use $f_y = 420$ MPa. (60000 psi).

- a- Slab with beams (8 \times 6) m clear span with α_m = 2.2
- b- Slab without drop panels (5.5 \times 4) m clear span with α_m = 0.11
- c- Flat plate (5×4.5) m clear span.
- d- Flat slab with drop panels (6×6) m clear span.
- e- Slab with beams (5.8 \times 5.2) m clear span with α_m = 1.5

Solution:

a- Slab with beams (8 \times 6) m clear span with α_m = 2.2

$$\alpha_{\rm m} = 2.2 > 2.0$$

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \quad \beta = \frac{\ell_n}{S_n} = \frac{8}{6} = 1.33$$

$$h = \frac{8000 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.33} = 183.4 \text{ mm} > 90 \text{ mm O.K}$$

Use h \approx 190 mm

b- Slab without drop panels (5.5 \times 4) m clear span with $\alpha_m {=}~0.11$

$$\alpha_{\rm m}$$
= 0.11 < 0.2 go to ACI Code Table 8.3.1.1
h = $\frac{\ell_n}{33} = \frac{5500}{33} = 166.67$ mm > 125 mm O.K
Use h ≈ 170 mm ■

c- Flat plate (5×4.5) m clear span.

h =
$$\frac{\ell_n}{33}$$
 = $\frac{5000}{33}$ = 151.5 mm > 125 mm O.K
Use h ≈160 mm ■

d- Flat slab with drop panels (6×6) m clear span.

h =
$$\frac{\ell_n}{36} = \frac{6000}{36} = 166.67 \text{ mm} > 100 \text{ mm O.K}$$

Use h ≈ 170 mm ■

e- Slab with beams (5.8 \times 5.2) m clear span with α_m = 1.5

$$0.2 < \alpha_{\rm m} = 1.5 < 2.0$$

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{1400} \right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \quad \beta = \frac{\ell_n}{S_n} = \frac{5.8}{5.2} = 1.115$$
$$h = \frac{5800 \times \left(0.8 + \frac{420}{1400} \right)}{36 + 5 \times 1.115 \times (1.5 - 0.2)} = 147.5 \text{ mm} > 125 \text{ mm O.K}$$

Use $h \approx 150 \text{ mm} \blacksquare$

Example 4: Find the minimum thickness of a slab for an interior panel due to deflection control for the following: Use $f_y = 350$ MPa

a. Slab with beams (8.1 × 8.2) m clear span with $\alpha_{\rm fm}$ =2.3

b. Flat plate (4.4×4.6) m clear span.

c. Flat slab with drop panels (6.2×6.2) m clear span.

Solution:

a. Slab with beams (8.1 \times 8.2) m clear span with $\alpha_{\rm fm}{=}2.3$

: $\alpha_{fm}=2.3>2$ then ACI Table 8.3.1.2 shall be used

h =
$$\frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$
 or 90 mm
 $\beta = \frac{\ell_n}{s_n} = \frac{8200}{8100} = 1.012$
h = $\frac{8200 \times \left(0.8 + \frac{350}{1400}\right)}{36 + 9 \times 1.012} = 190.87$ mm > 90 mm O.K.
∴ h ≈ 200 mm ■

b. Flat plate (4.4×4.6) m clear span.

From ACI Table 8.3.1.1

Table 8.3.1.1—Minimum thickness of nonpre-
stressed two-way slabs without interior beams
(mm) ^[1]

		Witho	Without drop panels ^[3]			ı drop pan	els ^[3]		
		F ()			Interior				Interior
		Exterio	Exterior panels		Exterior	r panels With	panels		
	fy, MPa ^[2]	edge beams	edge beams ^[4]	 	edge beams	edge beams ^[4]			
	280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	ℓ _n /36	<i>ℓ</i> _n /40	<i>ℓ</i> _n /40		
$f_y = 350 \text{ MPa}$	420	<i>ℓ</i> _n /30	<i>ℓ</i> _n /33	ℓ _n /33	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36		
	520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34		

For
$$f_y = 280$$
 MPa $h = \frac{\ell_n}{36}$
For $f_y = 420$ MPa $h = \frac{\ell_n}{33}$
For $f_y = 350$ MPa $h = \frac{\ell_n}{34.5}$ (by linear interpolation)
 $h = \frac{\ell_n}{34.5} = \frac{4600}{34.5} = 133.33$ mm > 125 mm O.K
 $\therefore h = 140$ mm

c. Flat slab with drop panels (6.2×6.2) m clear span. From ACI Table 8.3.1.1

Table 8.3.1.1—Minimum	thickness of nonpre-
stressed two-way slabs	without interior beams
(mm) ^[1]	

		Witho	Without drop panels ^[3]			With drop panels ^[3]		
		Exterior panels panels		Interior panels	Exterio	r panels	Interior panels	
	fy, MPa ^[2]	Without edge beams	With edge beams ^[4]		Without edge beams	With edge beams ^[4]	+	
f 350 MPa	280	<i>ℓ</i> _n /33	<i>ℓ</i> _n /36	<i>ℓ</i> _n /36	ℓ _n /36	<i>ℓ</i> _n /40	<i>ℓ</i> _n /40	
iy = 550 ivii a	420	<i>ℓ</i> _n /30	<i>ℓ</i> _n /33	<i>ℓ</i> _n /33	ℓ _n /33	<i>ℓ</i> _n /36	ℓ _n /36	
	520	<i>ℓ</i> _n /28	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /31	<i>ℓ</i> _n /34	<i>ℓ</i> _n /34	

For $f_y = 280$ MPa $h = \frac{\ell_n}{40}$ For $f_y = 420$ MPa $h = \frac{\ell_n}{36}$ For $f_y = 350$ MPa $h = \frac{\ell_n}{38}$ (by linear interpolation) $h = \frac{\ell_n}{38} = \frac{6200}{38} = 163.15$ mm > 100 mm O.K $\therefore h = 170$ mm **Example 5:** Find the minimum thickness of a slab for an interior panel due to deflection control for the following: Use $f_y= 280$ MPa (40000 psi)

a. Slab (2.5 × 3.5) m clear span with $\alpha_{fm}=3.2$

b. Flat plate (6.5×7.0) m clear span.

- c. Slab without drop panels (6.5 \times 5.5) m clear span with α_{fm} =0.12
- d. Slab with drop panels (6.0 \times 7.5) m clear span with α_{fm} =0.15

Solution:

a. Slab (2.5 \times 3.5) m clear span with $\alpha_{\rm fm}{=}3.2$

: α_{fm} = 3.2 > 2 then **ACI Table 8.3.1.2** shall be used

h =
$$\frac{\ell_n \times \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$
 or 90 mm
 $\beta = \frac{\ell_n}{s_n} = \frac{3500}{2500} = 1.4$
h = $\frac{3500 \times \left(0.8 + \frac{280}{1400}\right)}{36 + 9 \times 1.4} = 72.01$ mm < 90 mm not O.K.
∴ h = 90 mm ■

b. Flat plate (6.5×7.0) m clear span.

From ACI Table 8.3.1.1 $h = \frac{\ell_n}{36} = \frac{7000}{36} = 194.44 \text{ mm} > 125 \text{ mm O.K.}$ Use $h \approx 200 \text{ mm}$

c. Slab without drop panels (6.5 \times 5.5) m clear span with $\alpha_{fm}{=}$ 0.12

$$\therefore$$
 α_{fm}= 0.12 < 0.2 then **ACI Table 8.3.1.1** shall be used
h = $\frac{\ell_n}{36} = \frac{6500}{36} = 180.55$ mm > 125 mm O.K.
Use h ≈ 190 mm ■

d. Slab with drop panels (6.0 × 7.5) m clear span with α_{fm} = 0.15 $\approx \alpha_{fm}$ = 0.15 < 0.2 then ACI Table 8.3.1.1 shall be used

h =
$$\frac{\ell_n}{40} = \frac{7500}{40} = 187.5 \text{ mm} > 100 \text{ mm O.K.}$$

Use h ≈ 190 mm ■

Example 6: Find the minimum thickness of slabs due to deflection control for the following:

- 1. Interior panel without drop panel (3 \times 2.5) m clear span with α_m = 0.12, f_y = 420 MPa
- 2. Exterior panel, flat slab with drop panel without edge beam (6.32 \times 5.25) m clear span, f_y = 420 MPa
- 3. Exterior panel, flat plate (6.45 \times 5.15) m with edge beam, $f_y = 350$ MPa
- 4. Exterior panel (6.45 \times 4.15) m with α_m = 1.3 and for edge beam α = 0.3, f_y = 400 MPa

Solution:

1. Interior panel without drop panel (3 \times 2.5) m clear span with α_m = 0.12, f_y = 420 MPa

Since $\alpha_m = 0.12 < 0.2$ then **ACI Table 8.3.1.1** shall be applied

$$h = \frac{\ell_n}{33} = \frac{3000}{33} = 90.9 \text{ mm} < 125 \text{ mm not O.K}$$

Use h = 125 mm

2. Exterior panel, flat slab with drop panel without edge beam (6.32 \times 5.25) m clear span, f_y = 420 MPa

h = $\frac{\ell_n}{33} = \frac{6320}{33} = 191.5 \text{ mm} > 100 \text{ mm O.K}$ Use h $\approx 200 \text{ mm}$

3. Exterior panel, flat plate (6.45 \times 5.15) m with edge beam, $f_y = 350$ MPa

For
$$f_y = 280$$
 MPa $h = \frac{\ell_n}{36}$
And for $f_y = 420$ MPa $h = \frac{\ell_n}{33}$

• Since the value of $f_y = 350$ MPa is lying in the mid between the above two values, therefore factor in the dominator will be (33 + 36)/2 = 34.5

:
$$h = \frac{\ell_n}{34.5} = \frac{6450}{34.5} = 186.9 \text{ mm} > 125 \text{ mm O.K}$$

Use $h \approx 190 \text{ mm}$

4. Exterior panel (6.45 \times 4.15) m with α_m = 1.3 and for edge beam α = 0.3, f_y = 400 MPa

Since $\alpha_m = 1.3$ ACI Table 8.3.1.2 shall be applied

• α_f for edge beam 0.3 < 0.8, then the equation shall be increased by 10 percent.

h = 1.1 ×
$$\frac{\ell_n \left(0.8 + \frac{r_y}{1400}\right)}{36 + 5\beta(\alpha_{\rm fm} - 0.2)}$$
 $\beta = \frac{\ell_n}{s_n} = \frac{6.45}{4.15} = 1.554$

h = $\frac{6450 \times \left(0.8 + \frac{400}{1400}\right)}{36 + 5 \times 1.554 \times (1.3 - 0.2)}$ = 172.9 mm > 125 mm O.K Use h ≈ 180 mm ■

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